Journal of Maritime Research, Vol. III. No. 1, pp. 27-40, 2006

ISSN: 1697-4840

ODD LEVEL SET: A NEW METHOD FOR SIMULATION OF FREE SURFACE PROBLEMS

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Abstract

This paper introduces a new stabilized finite element method based on FIC (Oñate and García, 2001; Oñate et al, 2004; García et al, 2005) and ALE techniques (Hirt et al, 1974), specially developed for analysis of naval hydrodynamics problems. The main innovation of this method is the application of domain decomposition concept in the statement of the problem, in order to increase accuracy in the capture of free surface as well as in the resolution of governing equations in the interface between the two fluids. Free surface capturing is based on the solution of a level set equation, while Navier Stokes equations are solved using an iterative monolithic predictor-corrector algorithm (Codina, 2001), where the correction step is based on the imposition of the divergence free condition in the velocity field by means of the solution of a scalar equation for the pressure. In this paper an application of the new methodology to the simulation of roll movement in a real geometry of a high speed craft is presented.

Key words: Finite Element Method (FEM), Free Surface, Naval Hydrodynamics, Navier Stokes, Level Set, Domain Decomposition.

STATEMENT OF THE PROBLEM

The velocity and pressure fields of two incompressible and immiscible fluids moving in the domain $\Omega \subset \boldsymbol{R}^d$ (d=2,3) can be described by the incompressible

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Navier Stokes equations for multiphase flows, also known as non-homogeneus incompressible Navier Stokes equations (Lions, 1996):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u) = 0$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho u_i u_j\right) + \frac{\partial p}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} = \rho f_i \tag{1}$$

$$\frac{\partial u_i}{\partial x_i} = 0$$

Where $1 \leq i, \leq j\, d$, ρ is the fluid density field, u_i is the ith component of the velocity field u in the global reference system x_i, p is the pressure field and τ is the viscous stress tensor defined by:

$$\tau_{ij} = \mu\left(\partial_i u_j + \partial_j u_i\right) \tag{2}$$

where μ is the dynamic viscosity.

Let $\Omega_1 = \{x \in \Omega | x \in \text{Fluid1}\}$ be the part of the domain Ω occupied by the fluid number 1 and let $\Omega_2 = \{x \in \Omega | x \in \text{Fluid2}\}$ be the part of the domain Ω occupied by fluid number 2. Therefore Ω_1, Ω_2 are two disjoint subdomains of Ω. Then

$$\Omega = \text{int}\left(\overline{\Omega_1 \cap \Omega_2}\right) \tag{3}$$

The system of equations (1) must be completed with the necessary initial and boundary conditions, as shown below.

It is usual in the literature to consider that the first equation of the system (1) is equivalent to impose a divergence free velocity field (the third equation in (1)), since the density is taken as a constant. However, in the case of multiphase incompressible flows, density can not be consider constant in $\Omega \times (0, T)$. Actually, it is possible to define ρ, μ fields as follows:

$$\rho, \mu = \begin{cases} \rho_1, \mu_1 & x \in \Omega_1 \\ \rho_2, \mu_2 & x \in \Omega_2 \end{cases} \tag{4}$$

Let $\psi: \Omega \times (0,T) \rightarrow \mathbb{R}$ be a function, in below named Level Set function, defined as follows:

$$\psi(x,t) = \begin{cases} d(x,t) & x \in \Omega_1 \\ 0 & x \in \Gamma \\ -d(x,t) & x \in \Omega_2 \end{cases} \tag{5}$$

where $d(x,t)$ is the distance to the interface between the two fluids, denoted by Γ, of the point x in the time instant *t*. From definition (5) it is trivially obtained that:

$$\Gamma = \{x \in \Omega \,|\, \psi(x,\cdot) = 0\} \tag{6}$$

Since the level set 0 identify the free surface between the two fluids, the following relations can be obtained:

$$n(x,t) = \nabla \psi\big|_{(x,t)} \;\; ; \;\; \kappa(x,t) = \nabla \cdot \left(n(x,t)\right) \tag{7}$$

where n is the normal vector to the interface Γ, oriented from fluid 1 to fluid 2 and κ is the curvature of the free surface. In order to obtain relations (7) it has been assumed that function ψ defined in (5) accomplish (Osher and Sethian, 1988; Osher and Fedkiw, 2001; Fedkiw et al, 1999):

$$\|\nabla \psi\| = 1 \qquad \forall (x,t) \in \Omega \times (0,T) \tag{8}$$

Therefore, it is possible to re-write definition (4) as follows:

$$\rho, \mu = \begin{cases} \rho_1, \mu_1 & \psi > 0 \\ \rho_2, \mu_2 & \psi < 0 \end{cases} \tag{9}$$

Let us write the density fields in terms of the level set function ψ as

$$\rho(x,t) = \rho(\psi(x,t)) \quad \forall (x,t) \in \Omega \times (0,T) \tag{10}$$

Then, density derivatives can be written as

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial \psi}\frac{\partial \psi}{\partial t}, \quad \frac{\partial \rho}{\partial x_i} = \frac{\partial \rho}{\partial \psi}\frac{\partial \psi}{\partial x_i} \tag{11}$$

Inserting relation (11) in the first equation of the system (1) gives

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_i}(\rho u_i)\underset{\frac{\partial u_i}{\partial x_i}=0}{=}\frac{\partial \rho}{\partial t}+u_i\frac{\partial \rho}{\partial x_i}=\frac{\partial \rho}{\partial \psi}\frac{\partial \psi}{\partial t}+\frac{\partial \rho}{\partial \psi}u_i\frac{\partial \psi}{\partial x_i}=\frac{\partial \rho}{\partial \psi}\left[\frac{\partial \psi}{\partial t}+u_i\frac{\partial \psi}{\partial x_i}\right]=0 \quad (12)$$

What gives as a result that the multiphase Navier Stokes problem (1) are equivalent to solve the following system of equations:

$$\begin{aligned}\frac{\partial \rho u_i}{\partial t}+\frac{\partial}{\partial x_j}\left(\rho u_i u_j\right)+\frac{\partial p}{\partial x_i}-\frac{\partial \tau_{ij}}{\partial x_j}&=\rho f_i\\ \frac{\partial u_i}{\partial x_i}&=0\end{aligned} \quad (13)$$

coupled with the equation

$$\frac{\partial \psi}{\partial t}+u_i\frac{\partial \psi}{\partial x_i}=0 \quad (14)$$

Equation (14) defines the transport of the level set function due to the velocity field obtained by solving (13).

As a conclusion, the free surface capturing problem can be described by equations (13) and (14). In this formulation, the interface between the two fluids is defined by the level set 0 of ψ.

It is possible to demonstrate, assuming the variables of the problem as sufficiently smooth, that the system (1) or equivalently the system given by equations (13) and (14) has a unique global solution (Lions, 1996).

Denoting by over-bar the prescribed values, the boundary conditions of problem (13) y (14) to be considered are

$$\begin{aligned}&u=\bar{u} \quad in \quad \Gamma_u\\ &p=\bar{p}, \quad n_j\tau_{ij}=\bar{t}_i \quad in \quad \Gamma_p\\ &u_j n_j=\bar{u}_n, \quad \left.\begin{aligned} n_j\tau_{ij}g_i&=\bar{t}_1\\ n_j\tau_{ij}s_i&=\bar{t}_2\end{aligned}\right\} \quad in \quad \Gamma_\tau\end{aligned} \quad (15)$$

Where the boundary $\partial\Omega$ of the domain Ω has been split in three disjoint sets: Γ_u, Γ_p where the Dirichlet and Neumann boundary conditions are imposed and Γ_τ

where the Robin conditions for the velocity are set. In above vectors g, s span the space tangent to Γ_τ. In a similar way, the boundary conditions for (14) are defined

$$\psi = \bar{\psi} \quad in \quad \Gamma_u \tag{16}$$

Finally, initial conditions for the problem to be considered are

$$u = u_0 \quad on \quad \Omega, \qquad \psi = \psi_0 \quad in \quad \Omega \tag{17}$$

where $\Gamma_0 = \{x \in \Omega | \psi_0(x) = 0\}$ defines the initial position of the free surface between the two fluids.

FIC STABILIZED PROBLEM

It is well known that the finite element (FEM) solution of the incompressible Navier-Stokes equations may suffer from numerical instabilities from two main sources. The first is due to the advective character of the equations which induces oscillations for high values of the velocity. The second source has to do with the mixed character of the equations which limits the stability of the solution to the satisfaction of the well known inf-sup condition. The stabilization technique used in this work is based on the FIC (Finite Incremental Calculus) method presented in (Oñate and García, 2001; Oñate et al, 2004; García et al, 2005; Oñate et al, 2006).

The stabilized FIC form of the governing differential equations (13) and (14) can be written as

$$r_{m_i} - \underline{\frac{1}{2} h_{m_{ij}} \frac{\partial r_{m_i}}{\partial x_j}} = 0, \quad r_d - \underline{\frac{1}{2} h_j \frac{\partial r_d}{\partial x_j}} = 0, \quad r_\psi - \underline{\frac{1}{2} h_j \frac{\partial r_\psi}{\partial x_j}} = 0 \tag{18}$$

The boundary conditions for the stabilized problem are written as

$$\begin{aligned} u &= \bar{u} \quad in\, \Gamma_u \\ p = \bar{p} \quad n_j \tau_{ij} - \underline{\frac{1}{2} h_j n_j r_{m_i}} &= \bar{t}_i \quad in\, \Gamma_u \\ u_j n_j = \bar{u}_n, \quad n_j \tau_{ij} g_i - \underline{\frac{1}{2} h_j n_j r_{m_i} g_i} &= \bar{t}_1 \quad in\, \Gamma_u \\ , \quad n_j \tau_{ij} s_i - \underline{\frac{1}{2} h_j n_j r_{m_i} s_i} &= \bar{t}_2 \end{aligned} \tag{19}$$

The underlined terms in equations (18) and (19) introduce the necessary stabilization for the numerical solution of the Navier Stokes problem (Oñate and García, 2001; Oñate et al, 2004; Oñate et al, 2006).

Note that terms r_{m_i}, r_d y r_ψ, y denote the residual of equations (13) and (14), this way, as example:

$$r_{m_i} = \frac{\partial \rho u_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho u_i u_j\right) + \frac{\partial p}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} - \rho f_i \tag{20}$$

The characteristic length distances h_j represent the dimensions of the finite domain where balance of mass and momentum is enforced. Details on obtaining the FIC stabilized equations and recommendation for the calculation of the stabilization terms can be find in (Oñate and García, 2001; Oñate et al, 2004; Oñate et al, 2006).

OVERLAPPING DOMAIN DECOMPOSITION

Let us consider next domain decomposition of domain Ω into three disjoint sub domains Ω_3, Ω_4, and Ω_5 in such a way that $\Omega_3 = \bigcup_e \Omega_3^e$, $\Omega_5 = \bigcup_e \Omega_5^e$.

Where Ω_3^e are the elements of the finite element partition $\Omega = \bigcup_e \Omega^e$, such as $\forall x \in \Omega_3^e \big| \psi > 0$ and Ω_5^e are the elements of the finite element partition such as $\forall x \in \Omega_5^e \big| \psi < 0$. The geometrical domain decomposition is completed with

$$\Omega_4 = \Omega \setminus \left(\Omega_3 \cup \Omega_5\right) \tag{21}$$

From this partition let us define two overlapping domains $\tilde{\Omega}_1$ y $\tilde{\Omega}_2$(see Figure 1):

$$\tilde{\Omega}_1 := \operatorname{int}\left(\overline{\Omega_3 \cup \Omega_4}\right), \quad \tilde{\Omega}_2 := \operatorname{int}\left(\overline{\Omega_4 \cup \Omega_5}\right) \tag{22}$$

Let V_h and Q_h be $V_{h,i} := \left\{ v \in V_h \middle| v_{|\partial\Omega \cap \partial\tilde{\Omega}_i} = 0 \right\}$, $Q_{h,i} := \left\{ v \in Q_h \middle| v_{|\partial\Omega \cap \partial\tilde{\Omega}_i} = 0 \right\}$ the finite element spaces to interpolate velocity and pressure field, respectively.

We propose to solve a modified problem of the original FIC stabilized system, written as follows: Given two velocity fields $u_{h,1}^n \in V_{h,1}$, $u_{h,2}^n \in V_{h,2}$, and two pressure fields $p_{h,1}^n \in Q_{h,1}$, $p_{h,2}^n \in Q_{h,2}$, defined in the overlapping sub domains $\tilde{\Omega}_1$ y $\tilde{\Omega}_2$, respectively, in the time t^n and a guess for the unknowns at an

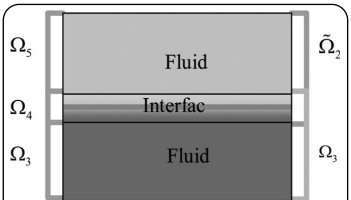

Figure 1. Geometric Decomposition of analysis Domain.

iteration $i-1$ at time t^{n+1}, find $u_{h,1}^{n+1,i} \in V_{h,1}$, $u_{h,2}^{n+1,i} \in V_{h,2}$ and $p_{h,1}^{n+1,i} \in Q_{h,1}$, $p_{h,2}^{n+1,i} \in Q_{h,2}$ in time t^{n+1}, by solving next discrete variational problem:

$$\begin{cases} \rho_1 \int_{\tilde{\Omega}_1} \frac{u_{h,1}^{n+1,i} - u_{h,1}^{n}}{\theta \cdot \delta t} \cdot v_h d\Omega + \rho_1 \int_{\tilde{\Omega}_1} \left(u_{h,1}^{n+\theta,i-1} \cdot \nabla\right) u_{h,1}^{n+\theta,i} \cdot v_h d\Omega - \int_{\tilde{\Omega}_1} \nabla \cdot v_h \cdot p_{h,1}^{n+\theta,i} d\Omega + \\ + \int_{\tilde{\Omega}_1} \nabla v_h \cdot \tau_{ij,1} \cdot d\Omega + \sum_{e=1}^{n_e} \int_{\tilde{\Omega}_1^e} \frac{1}{2} (h_m \cdot \nabla) v_h \cdot r_{m,1} d\Omega = \int_{\tilde{\Omega}_1} b \cdot v_h d\Omega + \\ + \int_{\Gamma_P \cap \partial\tilde{\Omega}_1} t \cdot v_h d\Omega + \int_{\Gamma_\tau \cap \partial\tilde{\Omega}_1} \left(t_1 g_1 + t_2 g_2\right) \cdot v_h d\Omega \\ \rho_1 \int_{\tilde{\Omega}_1} q_h \nabla \cdot u_{h,1}^{n+\theta,i} d\Omega + \sum_{e=1}^{n_e} \int_{\tilde{\Omega}_1^e} \frac{1}{2} (h \cdot \nabla) q_h \cdot r_{d,1} d\Omega = 0 \\ \left(u_1(x,0), v_h\right)_{\tilde{\Omega}_1} = \left(u_0(x), v_h\right)_{\tilde{\Omega}_1} \\ u_{h,1}^{n+1} = u_{h,2}^{n+1},\ p_{h,1}^{n+1,i} = n\left(\tau_{h,1}^{n+1,i} - \tau_{h,2}^{n+1}\right) n + p_{h,2}^{n+1} - \sigma\kappa \quad en\ \Gamma \end{cases} \tag{23}$$

$$\begin{cases} \rho_2 \int_{\tilde{\Omega}_2} \frac{u_{h,2}^{n+1,i} - u_{h,2}^{n}}{\theta \cdot \delta t} \cdot v_h d\Omega + \rho_2 \int_{\tilde{\Omega}_2} \left(u_{h,2}^{n+\theta,i-1} \cdot \nabla\right) u_{h,2}^{n+\theta,i} \cdot v_h d\Omega - \int_{\tilde{\Omega}_2} \nabla \cdot v_h \cdot p_{h,2}^{n+\theta,i} d\Omega + \\ + \int_{\tilde{\Omega}_2} \nabla v_h \cdot \tau_{ij,2} \cdot d\Omega - \int_{\tilde{\Omega}_2} v_h \cdot b \cdot d\Omega + \sum_{e=1}^{n_e} \int_{\tilde{\Omega}_2^e} \frac{1}{2} (h_m \cdot \nabla) v_h \cdot r_{m,1} d\Omega = \\ = \int_{\Gamma} n \cdot (\tau - p + \sigma\kappa) \cdot v d\Omega + \int_{\tilde{\Omega}_2} b \cdot v_h d\Omega + \int_{\Gamma_P \cap \partial\tilde{\Omega}_2} t \cdot v_h d\Omega + \int_{\Gamma_\tau \cap \partial\tilde{\Omega}_2} \left(t_1 g_1 + t_2 g_2\right) \cdot v_h d\Omega \\ \rho_2 \int_{\tilde{\Omega}_2} q_h \nabla \cdot u_{h,2}^{n+\theta,i} d\Omega + \sum_{e=1}^{n_e} \int_{\tilde{\Omega}_2^e} \frac{1}{2} (h \cdot \nabla) q_h \cdot r_{d,2} \cdot d\Omega = 0 \\ \left(u_2(x,0), v_h\right)_{\tilde{\Omega}_2} = \left(u_0(x), v_h\right)_{\tilde{\Omega}_2} \end{cases} \tag{24}$$

For $i = 1,2,3,\ldots$ until convergence, that is to say, until

$$\left\| u_h^{n+1,i+1} - u_h^{n+1,i} \right\|_\infty < tol_u, \quad \left\| p_h^{n+1,i+1} - p_h^{n+1,i} \right\|_\infty < tol_p \tag{25}$$

Where tol_u y tol_p are fixed tolerances.

It is possible to demonstrate that problem (23)-(24) is equivalent to (13)-(14) (Quarteroni and Valli, 1999).

Additionally, it is important to note that the proposed domain decomposition technique, allows to impose boundary conditions on the free surface, and therefore to take into account effects such as the surface tension in the interface, defined by

$$\left[-pI+\mu\tau\right]_{\Gamma}\cdot n=-\sigma\kappa\cdot n \tag{26}$$

where $[\cdot]_{\Gamma}$ notes the pressure jump in the interface and is the surface tension constant of the problem.

The authors have proposed to christen to this new methodology, as a combination of domain decomposition and level set techniques: ODDLS (Overlapping Domain Decomposition Level Set).

ALE FORMULATION

It is of interest in many applications to consider the movement of some parts of the analysis domain. In the mobile parts of the domain is more convenient to use a Lagrangean formulation of the equations and update the spatial discretization every time step. While in the fixed areas of the analysis domain, it is more efficient to use the standard Eulerian formulation. This type of mixed formulation is called "Arbitrary Lagrangian-Eulerian" (ALE) technique (Hirt et al, 1974).

It is possible to obtain a more general formulation of equations (13) and (14) considering next definition of the material derivatives:

$$\frac{D(\cdot)}{Dt}=\frac{\partial(\cdot)}{\partial t}+\left(u_j-u_j^m\right)\frac{\partial(\cdot)}{\partial x_j} \tag{27}$$

Where u_j^m is the relative velocity between the local axes fixed to the fluid particle and the global reference of the problem. This way, it is possible to obtain the ALE formulation of the residuals in (18) as follows

$$\begin{aligned} r_{m_i}&=\rho\frac{\partial u_i}{\partial t}+\rho\left(u_j-u_j^m\right)\frac{\partial u_i}{\partial x_j}+\frac{\partial p}{\partial x_i}-\frac{\partial \tau_{ij}}{\partial x_j}-\rho f_i \\ r_d&=\frac{\partial u_i}{\partial x_i} \qquad\qquad 1\le i,j\le d \\ r_\psi&=\frac{\partial \psi}{\partial t}+\rho\left(u_j-u_j^m\right)\frac{\partial \psi}{\partial x_j}\end{aligned} \tag{28}$$

ADAPTATION OF THE ODDLS METHOD FOR SOLVING MONOPHASE FLOW

It is usual in naval applications to have only one fluid of interest (water). These applications involve biphase flows with density and viscosity rations about 1000 y a

75, respectively. It is important for these cases to adapt the ODDLS technique to solve monophase problems, reducing the computational cost and capturing the free surface with the necessary accuracy and maintaining the advantages of the proposed method. In this case, the computational domain is reduced to the nodes in the water plus those in the air being connected to the water interface. The later nodes are used to impose the pressure and velocity boundary conditions on the interface.

At a computational level, this modification is equivalent to solve problem(23), imposing (29) by means of a local least squares technique.

The proposed monophase adaptation of the ODDLS method has been used in the application example shown in the next section.

CASE OF STUDY

The example of application of the presented technique is the analysis of a high speed boat. The general characteristics of this boat appear in the following table:

Overall length	11.2 m
Molded beam	2.5 m
Displacement	4.9 t
Design velocity	40 kn

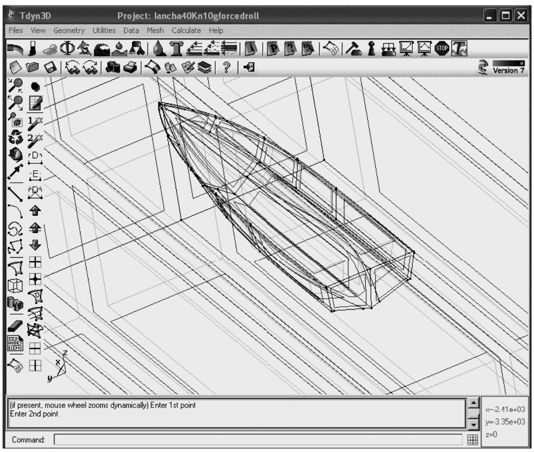

Figure 2. CAD Geometry of The boat Hull.

The geometry of the boat has been defined by means of NURBS patches by the designer, and later exported to GiD-Tdyn software (GiD; Tdyn), where we insert the necessary data for the analyses and mesh generation. The used geometry is displayed in Figure 2. In the mentioned program a computational domain of 45,8 m x 17.6 m x 11.5 m was generated. The referred volume was subdivided in two zones, an internal parallelepiped around the boat, of dimensions 32,8 m x 8,8 m x 6,5 m and an external domain, corresponding to the rest of the volume of analysis. These zones were used for the adaptation of the sizes of elements of the mesh to the analysis requirements. The objectives of this adaptation were two:

Reduce the element size in the neighbor zone of the boat to be able to capture the fluids dynamics phenomena of interest.

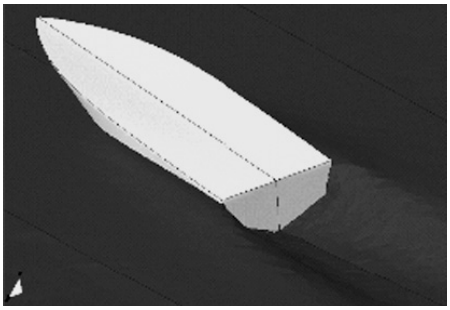

Figure 3. View of fluid Flow in the Stern (V=30 kn).

Create an external zone where the generated waves by the movement of the boat will be damped, reducing therefore the possible effects of bounces in the boundaries of the analysis domain.

This way a nonstructured mesh was generated in the zone near the boat with element size that vary between 0.05 m and.85 m. The resulting mesh of this process contains 420 000 linear tetrahedra, and it was used for all the simulations made in this work. The simulations have been made on salt water using real scale, ignoring the effect of the air in the resolution of the equations of the dynamics of fluids. Each one of the analyses consists of the following phases:

Initial phase: corresponds to the start-up of the simulation, beginning from rest point and arriving to towing speed. This phase is carried out with the ship fixed during 0.5 s of real time.

Towing phase: during 3,5 s of real time an analysis of the towing of the boat is carried out, leaving the boat free to trim and sink.

Rolling phase: during 11.5 s of physical time a forced roll of the boat in different conditions is carried out. Three different towing speeds have been analyzed: 20, 30 and 40 kn.

In the following table a comparative between the steady state situation obtained in the present work during the phase of towing and available experimental data for a scale model is presented:

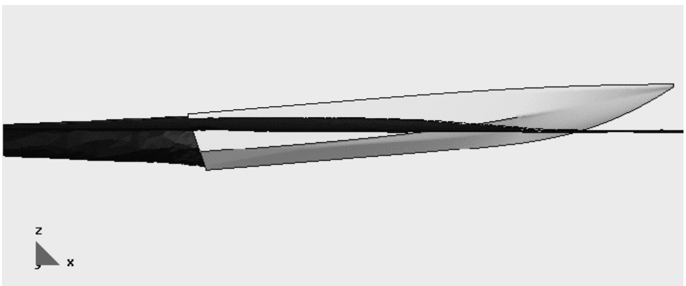

Figure 4. Lateral view of the free surface around the boat (V = 30 kn).

Velocity (kn)	Trim Angle (º)	
	Experimental (model scale)	This Work (Full scale)
20	8.3	7.4
30	5.8	6.3
40	4.1	4.5

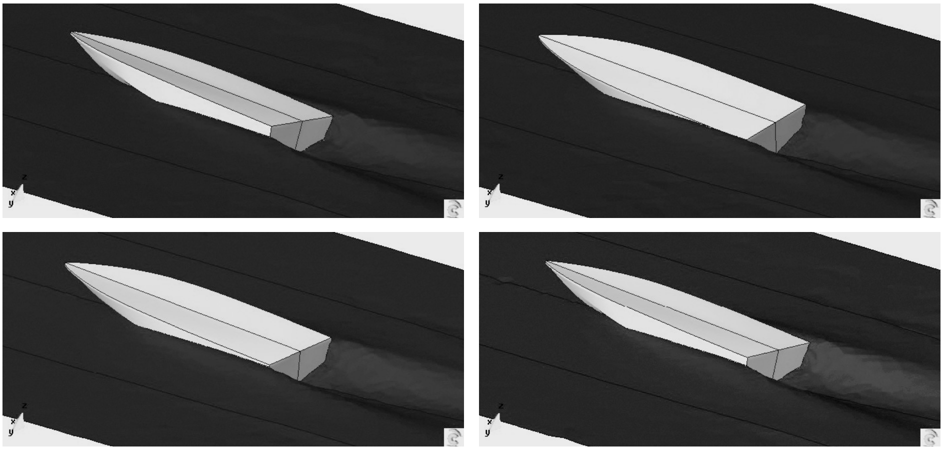

Figure 5. different view of the analysis results during a forced rolling of the boat (V = 30kn, θ = 10º).

Figure 3 shows the flow in the stern of the ship for the case of towing speed of 30 kn, whereas Figure 4 displays a lateral view of the free surface around the boat.

The rolling analyses were carried out for different amplitudes and a for a period near the resonance point of the movement, that has been estimated in T = 1.3 s.

The studied rolling amplitudes are 2, 5 and 10º for each towing speed. In the appendix there are different graphical results from the analyses carried out. Figure 5 shows results of the free surface for case V = 30 kn θ = 10º during a period of the forced balance of the boat. On the other hand, Figure 6 displays different images of the results of the speed field at a dimensionless distance y+ = 65 of the hull, throughout a forced roll period of the boat (case V =40kn θ = 10º). Where $y+ = y \cdot \rho \cdot u\tau / \mu$, being y the distance from the hull in the normal direction to the surface. And $u\tau$ the traction in the wall. Finally Figure 7 shows to different mesh cuts from the solution of the equation of level Set in the analysis case corresponding to V = 40 kn θ = 10º. In this image it is possible to appreciate how the method is able to capture the interface between air and water with sufficient accuracy, even with large elements.

The numerical results of these tests are presented in the following tables. In them the amplitude value of the moments induced by pressure and viscous effects are presented:

Amplitude of forced roll 2º

Velocity (kn)	Viscous force moment (N·m)	Pressure moment (N·m)
20	110	2.775
30	170	4.200
40	205	4.880

Amplitude of forced roll 5º

Velocity (kn)	Viscous force moment (N·m)	Pressure moment (N·m)
20	275	6.500
30	450	9.850
40	550	13.000

Amplitude of forced roll 10º

Velocity (kn)	Viscous force moment (N·m)	Pressure moment (N·m)
20	280	7.300
30	705	15.600
40	1.235	25.000

The results of the analyses show a clear influence of the speed in the moments that the fluid exerts on the boat.

From the two calculated components of the moment, the one corresponding to the integration of the pressure is practically in the same phase as the movement, being this contribution of the viscous efforts the unique one that causes the damping effect to the balance movement. As has already been seen in similar studies (García et al, 2005) the increase of the effect of the viscous forces is responsible for the increase of the damping effect of roll movement with the increase of boat speed. On other hand, it is important to note that moments due to pressures has a significant component due to dynamic effects. This is an expectable result due to the high velocity developed by the boat.

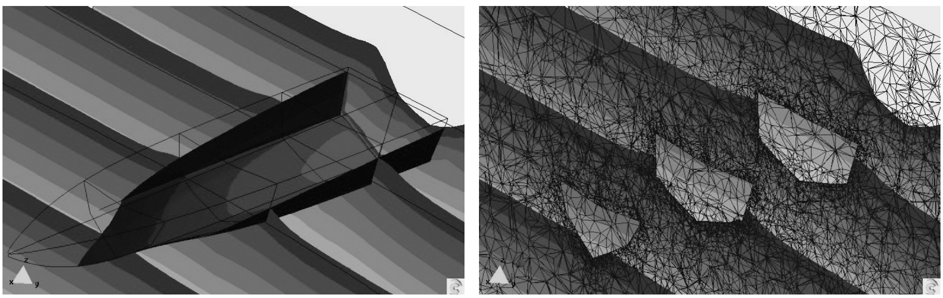

Figure 6. View of Level Set solution over the analysis mesh (level set) (V = 40 kn, θ = 10º)

CONCLUSIONS

The present work shows a new methodology for the analysis of problems with free surface denominated ODD Level Set. This methodology is based on application of domain decomposition techniques and allows increasing the accuracy of the free surface capturing (level set equation) as well as solving governing equations in the interface between water and air. The greater accuracy in the solution of the interface between the fluids allows the use of non-structured meshes, as well as the use of larger elements in the free surface. In addition, the method can be simplified by solving only one of the two fluids, which allows increasing the efficiency in those cases where the effect of one of the fluids can be neglected.

The proposed methodology has been integrated with an ALE algorithm for the treatment of the ship movement and has been applied in the analysis of the towing test and forced roll of a high speed boat. The satisfactory result of the qualitative analysis of the application study shows the capability of the presented methodology for studying this kind of problems.

ACKNOWLEDGEMENTS

The authors wish to thank to the company Nautatec for the authorization for the use of geometry and the experimental results of the high speed boat used as case of study of the proposed methodology.

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