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ON SCALING LAWS AND MARITIME TRANSPORT

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Abstract

Scaling, as a manifestation of the underlying general dynamics and geometry, is familiar throughout physics. It has helped scientists gain deeper insights into problems ranging across the entire spectrum of science and technology, as scaling laws typically reflect generic features and physical principles that are independent of the detailed dynamics or specific characteristics of particular models. Fluid mechanics and phase transitions are significant examples of physics in which scaling has illuminated important universal principles or structures, and has provided responses to practical problems. Also, complex systems as living organisms obey some scaling relations that capture these systems' essential features, if these do in fact exist. In contrast to the large diversity and complexity of living organisms, one finds the simplicity of the scaling behaviour of biological processes that holds true in a wide range of phenomena and a large range of energy and mass. The constructal theory states that flow systems evolve in time so that they develop the flow architecture that maximizes flow access under the constraints posed to the flow. This "extreme" principle has been quite successful in justifying allometric scaling laws, global circulation and climate characteristics, and even scaling effects in running, swimming and flying. Some of these moving relationships, the scaling between mass and speed, are tested in relation to ships and maritime transport, in which it is possible to find a reasonable continuity with the types of scales seen in living moving organisms, and some preliminary conclusions are drawn, pointing to the convenience but also to the difficulties of using large ships.

Keywords: Scaling laws, Complex systems, Ship speed, Ship size.

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INTRODUCTION

In complex systems, carefully designed decisions and guidelines might produce unexpected results because of particularities, or complex sets of reactions from residents or economic counterparts. Complexity tends to increase with size, such as when, for instance, larger ships or larger buildings are constructed, services tend to concentrate in large agglomerations, and transportation needs take on critical importance. Complex systems such as living organisms are known to follow approximate relationships as scaling laws between the variables that describe them. Some of these kinds of relationships are tested in relation to some questions on maritime transport, especially concerning ship size.

However, detailed physical relationships between the variables that determine these conditions are very complex due to the same complexity of the system and the multiple sets of variables needed to describe it, which are together considered as a physical system. Nevertheless, some ideas and tendencies seem to be shared by many modern ship transport systems: one should optimise such things as energy and time expenses for the transport of goods, and pollution issues. It then seems reasonable to expect some general trends (if not "exact" laws) that modern ships should follow because of these constrains, and it should be acceptable to think about it as complex, physical systems.

Then, we should learn from phenomenological relations in existing complex systems that have evolved over hundreds of millennia. As living organisms represent one of these systems, which have been largely studied, and are indeed a paradigm of complexity, studying general laws or general relationships in biology might give us an insight into our problem. In fact, it seems reasonable to conjecture that the coarse-grained behaviour of living systems might obey quantifiable universal laws that capture the systems' essential features, as (West and Brown, 2004) does.

As pointed out by (West and Brown, 2004), in biology the scaling observed is typically a simple power law: $Y = Y_0 M^b$, where Y is some observable magnitude, Y_0 a constant, and M is the mass of the organism. The exponent b usually approximates a simple fraction. Among the many fundamental variables that obey such scaling laws are metabolic rate, life span, heart rate, lengths of aortas and genomes, tree height, mass of cerebral grey matter, and others.

In fact, the constructal theory states that every flow system evolves in time so that it tends to develop the flow architecture that maximizes flow access under the constraints posed to the flow. It has been quite successful in justifying allometric scaling laws (Bejan, 2000), global circulation and climate characteristics (Reis and. Bejan, 2006), and even scaling effects in running, swimming and flying of animals (Bejan and Marden, 2006). However, as the boundaries and the conditions in complex systems might be not so evident, as (Burd, 2006) points for the case of traffic organisation in ant societies, care should be exercised in interpreting. We keep the main idea that even very complex systems, thus far as living organisms or man-produced systems, should follow some scaling laws if they have common basic principles in their dynamics or working phenomena. In fact, as (Isalgue et al., 2007) points out, transport conditioned complex systems as cities approach reasonably some of the living organisms scaling laws.

In this work, we look for dependency of cruising speed with mass for some home-made transportation devices, ships and planes, and compare them with sustained speeds for living organisms of different masses. We find a reasonable continuity of ships with the behaviour of swimming animals, even though with relatively reduced speeds for ships. This suggests that there is still an ample margin of speed which can be gained by ship transport.

SCALING LAWS IN RUNNING, FLYING, AND MOVING IN WATER

If we take into account that the economy/ecology principles are nearly the same everywhere, we can conjecture that an optimisation of resources in moving would imply a regular distribution of power among the different resistances to the movement (in walking, flying or swimming), for self-propulsating bodies. As many particularities exist in discontinuous jumping or in flying out of the horizontal, only speeds that can be sustained for a considerable time are to be taken into account.

In [(Bejan and Marden, 2006)] it is formulated a simplified model for walking and flying, which first assumes a composite movement of "pulsating" path, in which a muscular produced increase of height of the body centre of mass is followed by a "planning" decrease with air drag as source of resistance to movement. Assuming related movements (in vertical and in horizontal, in the field of gravity), form factors near 1, and drag coefficients also near 1, and equalizing the losses of potential energy and of air drag (in turbulent flow, as the usual case) per unit time, allows to obtain the speed v as function of the mass of the moving body M (both for running on ground or flying), giving the expression and the full line in fig. 1. Also, running on wheels could be considered a smoothing of the previous model, and the points (not designed) representing running cars are not very far (above) the continuous line in fig. 1. Railway transport lies somewhat below the line.

For objects moving in water, two sources of power loss are considered in (Bejan and Marden, 2006): viscous drag with water, and the need to increase the potential energy of the water displaced by the moving object (as nearly incompressible fluid, water has to move upwards to allow the pass of the object, this has been detected even for fish and for submarines). The latter point is equivalent to say "the existence of a wake", even in some cases it might be of very low amplitude and then, difficult to detect. Again, assuming form factors of the order of 1, coefficients of drag also near 1, and equalizing the energy losses per unit time of the two terms allows obtaining the relation of speed v with the mass M of the body moving on water, as the expression and the dotted line in fig. 1.

Data points referring animals in fig. 1 are obtained from (Bejan and Marden, 2006), data for ships refer to the contents of table I, and data for commercial airplanes refer to the contents of table II.

Table I: Data of mass/displacement, cruising speed and power for some ships.
The source where only date and page number is indicated refers to the journal (AINE).

Ship	Function	Displacement (metric tonnes)	Speed knots (m/s)	Power (kW)	Source
USS Alabama	Submarine (warship)	18000	25 (12,9)	45000	Wikipedia, 2008
USS A.Lincoln	Aircraft carrier	103000	>30 (15,4)	194000	
A. Bazan class	Frigate (warship)	6250	28 (14,4)	34800	
Marques de Comillas	Container slip	19053	17,7 (9,1)	8580	April 98, p. III ships
Jonas	Fishing	130	10,9 (5,6)	510	March 98, p. II ships
United Nadja	Tanker (chemical)	8197	13,2 (6,79)	2640	January 98, p. I ships
Stolt sea class	Tanker (chemical)	30243	15,2 (7,82)	6500	July 99, p. XV ships
Navion Britania	Tanker (oil)	145910	15 (7,71)	11520	February 99, p. X
Kica	Fishing	358	11 (5,66)	960	June 99, p. XIV
Skane	Ferry	23420	22 (11,32)	7240	December 98, p. IX
Hilder K/ Helgoland	Merchant	9200	15 (7,71)	3520	November 98, p. VIII
Quenn Mary II	Passenger ship	100000	30 (15,4)	117000	Wikipedia, 2008
HMS EndurancA171	Ice breaker	6000	14 (7,2)	6000	
Isla de Botafoc	Ferry	12000	18 (9,26)	12000	Balearia, 200
Alcantara	Fast ferry	940	38 (19,55)	20400	Acciona, 2008

Table II: Data of mass, cruising speed and power for some commercial airplanes.

Airplane	Mass (kg)	Speed (m/s)	Power (kW)	Source
B-747-B	440000	289	342000	Boeing, 2008
B-737	72100	246	44250	
B-717	51000	225	37080	
A-350-800	245000	289	192470	Airbus, 2008
A-340-600	368000	292	292400	
A-320	73500	242	55580	
Skyhawk	1300	62	110	Cessna, 2008
C-295	22000	133	33000	EADS, 2008

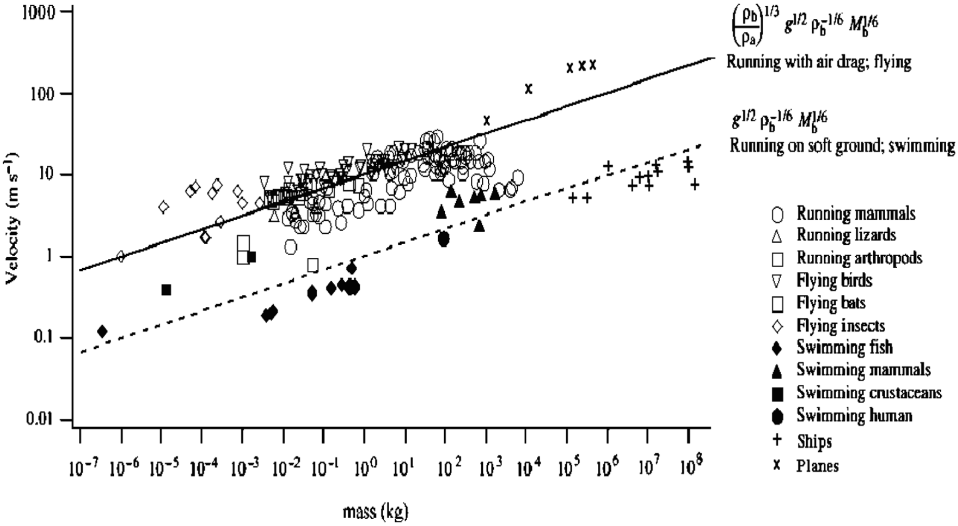

Figure 1. Speed (velocity) versus mass for different moving objects. Values for the straight lines: g, acceleration of gravity (9,8 m/s²); M_b, mass of the moving body; ρ_b, density of the moving body; ρ_a, density of air.

It should be noted that both calculated lines in figure 1 show a power dependence of speed v with mass M, with exponent equal to 1/6. As form factors have been assumed to be 1 or near 1, and density of moving bodies does not vary a lot, mass is approximately proportional to volume and then to representative length to the 3rd power, so speed shows a dependency on representative length to ½, remembering the known law of ships: “critical” speed in knots around square root of the length in feet.

Near 15 orders of magnitude (10^{15}) in mass are covered by data in fig. 1. Speed of moving organisms and moving objects cover only about a factor of 3000, but a clear tendency to follow the lines (continuous for running or flying, dotted for swimming) is observed. The speeds attainable for moving in air or running are higher because of the low density of air, which produces a reduced drag in the usual conditions (in turbulent flow, drag depends on: $\rho_{medium}\ v^2$). The detailed conditions of the travel (i.e., running, flying or going on wheels) appear smoothed by the main factors of air drag and pulsating (impact) centre of mass height change, taken as the most important facts in the model.

At low masses of the moving object (insects and crustacean), systematic deviations to higher speeds seem to be present. The reason might be that for small objects, the Reynolds number implied in the movement of the body is relatively low, and then the drag would be lower than assumed. It can be observed that large planes seem to perform very well, but this is an artifact, as the nominal speeds are usually obtained at high altitudes where the density of air is much lower (and then, drag is also much lower) than assumed in the computation of the dotted line in fig. 1.

War ships and fast ferries are the fastest ships. However, even using large amounts of power, they perform just near the dotted line. Also, even nuclear powered submarines do not travel much faster than other ships. The actual belief is that the wake is an important obstacle for increasing ship speed, the existence of a relatively strong wake for ships might seem a justification for relatively low performance of ships compared to some fish. However, the data shows that a nuclear submarine does not perform much better than ships, so it is suggested the real importance of some kind of wake, as the model for swimming assumes, in the form of the need to "expulse" water from the front of the moving object, even if the detection of a wake is more difficult as it becomes larger in extension but of very low amplitude.

Also, it is to be noted that war planes, which have not been represented, can go much faster that the continuous line would suggest, at expenses of very large amounts of power. Rockets are not represented, as they are used mostly for strategic reasons and not for considerable mass transport.

Another question is how do large "hunters", as dolphins or sharks, to perform considerably above the line for swimming in fig. 1. This challenging fact shows that it should be possible to improve ship speeds, maybe with hydrodynamic design, or even with new technologies not yet advised.

A further consideration on power has to be done. From the data in tables I and II, and the data for animals and cities in (Isalgue et al., 2007), we can obtain the figure 2, which shows a gross representation of power as function of mass for the different entities considered. The continuous line follows the observed dependency of metabolic power with mass for the animals, from allometric scaling law (West and Brown, 2004; Isalgue et al., 2007).

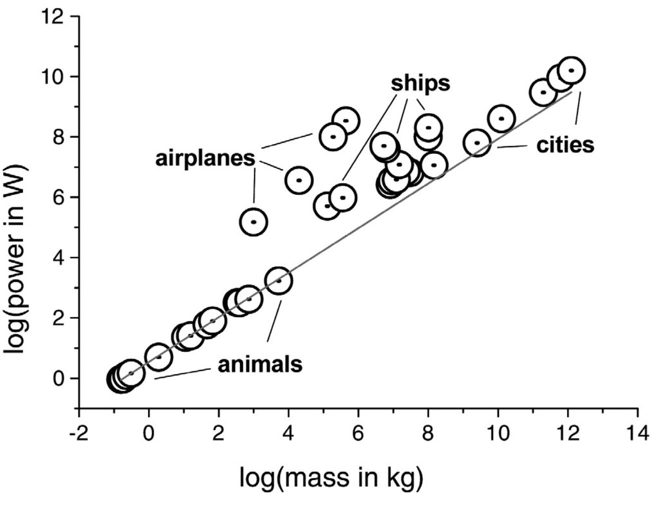

Figure 2. Power as a function of mass. Some animals, airplanes and ships are represented. Data for animals from (West and Brown, 2004), data for cities from (Isalgue et al., 2007) and data for ships and planes from tables I and II.

The different groups appear well separated: animals and cities appear to have approximately the same kind of dependence of power with mass, as (Isalgue et al., 2007) indicates.

Airplanes use lots of power compared to other entities of comparable mass, and the power used by airplanes seems to increase with mass stronger than for animals. The slope in the graph seems to approach 1, in place of the slope very near ¾ for animals [see (West and Brown, 2004; Isalgue et al., 2007) about the slope ¾] . Constructive reasons, and the fact that larger commercial airplanes flight higher than small planes, can be a reason for this.

For ships, large differences in power (a factor of near 100 times) might be encountered for the same mass, because very different ships (different missions) are plotted. Different ships might perform very differently, from a fast warship to a slow tanker. However, it has to be noted that "slow" large ships use comparatively very low amounts of power, near the continuous line, and even warships or fast ferries use much less power than would do an airplane of comparable mass (ships use power less than around 1/100 of equivalently massive airplane). Airplanes are faster, but its speed is only around 50 times higher than that of ships. Then, the result is the known fact that large ships, with large payload compared to its weight, are the best (economic) option for cargo transportation. From the analysis here, larger ships show a decrease in power per unit mass (power increases less than mass) and an increase in speed, so transport with large ships should be preferred. However, large ships ask for large facilities for cargo handling and for maintenance of the ships, and also ask for a large effort in logistics.

CONCLUSIONS

Complex systems as living organisms obey some scaling relations that capture these systems' essential features. The scaling behaviour has been attributed to the basic characteristics of these systems. An "extreme" principle, inscribed in the constructal theory, has been quite successful in justifying allometric scaling laws in living organisms, global circulation and climate characteristics, and even scaling effects in running, swimming and flying of animals. Some of these scaling in movement relationships, the scaling between the mass and cruising speed, has been tested here in relation to ships and maritime transport.

The cruising speed of commercial planes is somewhat higher than expected in comparison with the speed to mass relation for flying animals because the reduced density of air at high altitudes (and then, reduced drag). It has been found a reasonable continuity of the relation between speed and mass of ships with the types of scales seen in living moving organisms. The classical dependence of speed with size of a ship (i.e., $v \propto (\text{Length})^{1/2}$) might be recovered from some basic assumptions. Some preliminary conclusions might be drawn, suggesting some improvements in the technology might be able to obtain higher speeds for ships.

The challenging fact that large swimming "hunters", as dolphins or sharks, perform in speed clearly above a line (in fig. 1) designed with a model from (Bejan and Marden, 2006) for swimming, while ships perform generally below, shows that it should be possible to improve ship speeds, maybe with new hydrodynamic design, or even with new technologies not yet advised.

Some considerations on power have been also done, comparing animals, cities, ships and planes. The obtained graph (fig. 2) shows the strong convenience of large or very large ships, as power increases clearly slower than mass, and speed tends to increase with mass (thus reducing time of transport, and then, a diminishing of energy spent in transport is clear for large ships). This shows the convenience of large ships for economy cargo transport. The analysis of energy spent in transport is clearly favourable to the use of large ships. However, large ships produce the need of large facilities and terrestrial transports support to load and unload the ships (as well as of large facilities for maintenance of the ships), and this becomes usually a bottleneck in transport.

A further point to be considered, not done here, is the probability of occurrence and results of an accident, concerning very large loads and human and environmental impact of a loss if using very large ships. This might limit effectively the size of ships recommendable because of economic reasons in case of accident or loss.

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