Journal of Maritime Research, Vol. VII. No. 3, pp. 35-42, 2010

ISSN: 1697-4840

THE INFLUENCE OF THE STATIC PARTS MASS OF A MACHINE ON ITS NATURAL FREQUENCIES OF VIBRATION

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Received 14 April 2010; in revised form 27 May 2010; accepted 14 September 2010

Abstract

In this article it is intended to highlight the importance of choosing correctly the stationary parts mass in a machine to reduce the amplitude of the vibrations produced by moving parts of it, and in order to control its natural frequencies of vibration to try to avoid critical speeds during its normal performance. We have made three simulations, in which we have kept constant all parameters that influence in the natural frequency of the machine and we have changed one, the mass of the static parts of it.

Keywords: Machine, vibration, natural frequency, amplitude, unbalanced, damping.

INTRODUCTION AND ASSUMPTIONS

The problems generated by the engines and machines vibrations are several, but all of them have a common factor the machine lifetime reduction. For this reason it is so important understand how the problem is generated to try to find a more suitable solution for this particular problem.

Assuming that any machine is made with reciprocating and rotating moving elements and other static; the rotating parts balancing is not perfect, so they can be both statically and dynamically unbalanced (in the first the centre of gravity of the piece is

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outside the rotation axis, which generates a inertial force uncompensated; in the second the change of the angular momentum is not zero, producing a inertial torque uncompensated). The movement of parts with reciprocating motion also generates alternative inertial forces that induce the vibration of the whole machine.

This vibration is generated in a continuous medium, in this mass, friction and stiffness of the system are located along all parts, therefore the system has infinite degrees of freedom and infinite natural frequencies, which causes that mathematical analysis would be very complex. In order to resolve this mathematical problem; the best way to get a very approximate solution to the exact solution is to use iteration methods such as:

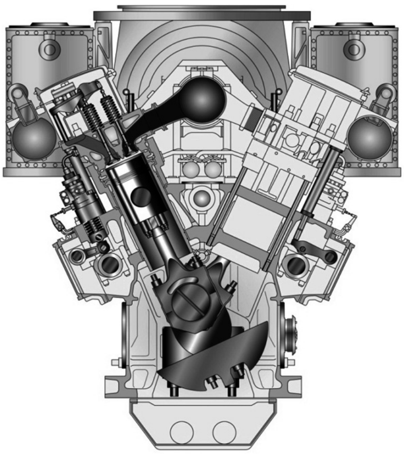

Figure 1. View of a generic machine.

Stodola method: it is an iterative process used to calculate of the main modes and natural frequencies of no-damped vibration systems. It is a physical mean and it does not need to derive the differential equation of the motion.

Matrix iteration: it is another iterative process that lead to the main modes of a vibration system and its natural frequencies, the displacements of the masses to be approximately, and base on these shifts it is obtained the system matrix equation, which is developed and finding the first vibration mode, and others are obtained by orthogonality principle.

Holzer method: it is a tabular method valid for any system, in which it is made the successive assumptions of the natural frequencies of the system which is based on the calculation of the frequencies calculated in the previous steps.

Another option is to simplify the system to one equivalent with fewer freedom degrees, which is easier to analyze, and of this way we can obtain similar results for both analyses being the last simpler than first.

MATERIAL, METHODS AND EQUATIONS

To simplify the system it is going to reduce all the parts that constituting the machine by a single mass and equivalent to all parts of the same, to calculate this mass it is used the notion of reduced mass. The distributed mass system with a rather complex motion (rotation, alternative or combined) is replaced by a mass concentrated at a point and a translational motion, this mass is called the reduced mass of the system in question, and this reduced mass is equivalent to the original with respect to kinetic energy "*T*".

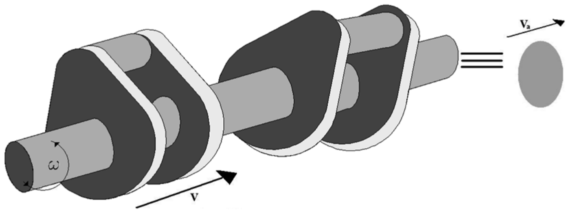

Figure 2. Identification of a system with a particle with mass.

To determine the value of this mass is necessary to identify the total kinetic energy of the machine to the mass of the particle *"A"*, for a machine this procedure is relatively simple since all machine parts have moving plane, a fact which greatly simplifies equation:

$$\boldsymbol{T_{mach}} = \sum_{i=0}^{n} {}^{1}\!/_{2}\, I_i \omega_i^{\,2} + \sum_{i=0}^{n} {}^{1}\!/_{2}\, m_i v_i^{\,2} \equiv {}^{1}\!/_{2}\, M v_a^{\,2}\ ; \qquad (1)$$

The kinetic energy of the machine "T_{mach}" is equal to the second term of the equation (1) and we identify this with the kinetic energy of a particle of mass "m_a" and velocity "v_a". To increase the simplifications in the calculation of "m_a" we assume that "v_a" is equal to 1 m/s, so the equation is simplified to the maximum.

The procedure to determine the system equivalent stiffness and damping coefficient it is similar to finding the reduced mass, to calculate the stiffness coefficient is used elastic energy of deformation of all parts of the machine with a elastic energy of a spring whose coefficient of stiffness will be *"K"*, for the equivalent damping coefficient is identifying all the energy dissipated by the system (internal friction, friction with the lubricant and bearings ...) with the energy dissipated by a damper whose constant was *"C"* is moving at a velocity "v_a".

DEVELOPMENT PHYSICAL-MATHEMATICAL

With these considerations is done to simplify the equation of motion of the continuous system by the equation of the equivalent system, with only one degree of freedom, which is assumed to exist displacement in only one direction (displacement in the other directions and rotations are obvious, without loss of generality) to further simplify the analysis; to obtain the equation using the principle of D'Alembert and assuming that the motion of the mass is simple harmonic and therefore its position has the form , in the light of free-body diagram we can see:

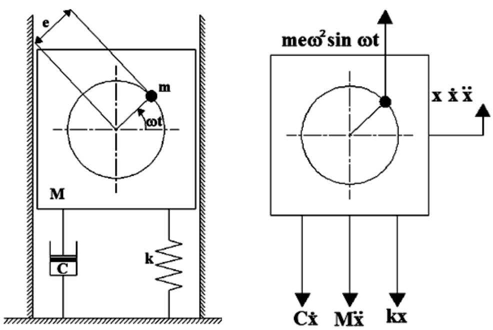

Figure 3. Problem to be studied and vector diagram of forces.

$$M\ddot{x} + c\dot{x} + kx = me\omega^2 \operatorname{sen} \omega t \qquad (2)$$

The meaning of each term is the next:

— $M\ddot{x}$: Is the inertial force that acting on the set of the machine.

— $M\dot{x}$: Is the damping force, by which the energy of the system is dissipated like heat.

— kx: Is the elastic deformation force.

— $ne\omega^2$ *sin* ωt: It represents the force generated by the vibration of the system, "m" is the unbalanced mass and "e" is the distance from the axis of rotation to "m", y "ω" is the frequency of the machine, this force can be centrifugal force or a first order force actuating on the reciprocating piston.

By theory of vibrations the solution to this equation for one variable "x" is an expression like:

$$x = \left[Ce^{-\zeta\omega nt} \operatorname{sen}(\omega dt + \phi)\right] + \left[\frac{me\omega^2}{\sqrt[2]{(k - M\omega^2)^2 + (C\omega)^2}} \operatorname{sen}(\omega t - \psi)\right] \qquad (3)$$

The first term of the right side of the equation (3) represents the response of the transient statement , which has no relative importance because it disappears with over time; the second term of that represents the steady state response of the system and whose analysis is really important to study to know the parameters of the vibration of the machine such as amplitude, frequency... Otherwise writing the second term in the expression above:

$$x = \frac{\frac{me\omega^2}{k}}{\sqrt[2]{\left[1 - \left(\frac{\omega}{\omega n}\right)^2\right]^2 + \left[2\zeta\left(\frac{\omega}{\omega n}\right)\right]^2}} \operatorname{sen}(\omega t - \psi) \qquad (4)$$

Where each term ha the usual meaning as defined in the theory of vibrations, but we will explain the most important "ω_n" the natural frequency of the system for this case is calculated as $\sqrt[2]{\boldsymbol{k/M}}$ is a fundamental parameter because it defines the resonance condition of the system, in this situation the amplitude of the vibration is only limited by damping factor "ζ" of the system, as this value is reduced vibration amplitude tends to infinity, hence the importance of choosing appropriate the values of: "k", "M" and "c" so the natural frequency of the system are far from the frequencies that excite the system "ω" (for a continuous medium that possesses infinite natural frequencies this conclusion, that the natural frequency must be far from the excitation frequencies, is the same).

From the above equation it also follow that if increased “M” (mass of the whole machine) for “m” (unbalanced mass that induces the vibration) the amplitude of vibration is reduced by reducing it also reduces the deformation of the pieces, what causes mechanical stress that occurs because of the vibration also decreases, since both are proportional (assuming that the parts are working as perfectly elastic, that is common practice).

DEVELOPMENT OF SIMULATED PROBLEM

The following figure represents a simulation of the proposed problem, to show that everything which follows from the theory of vibrations is met. This scheme is generated to simulate this problem is as follows:

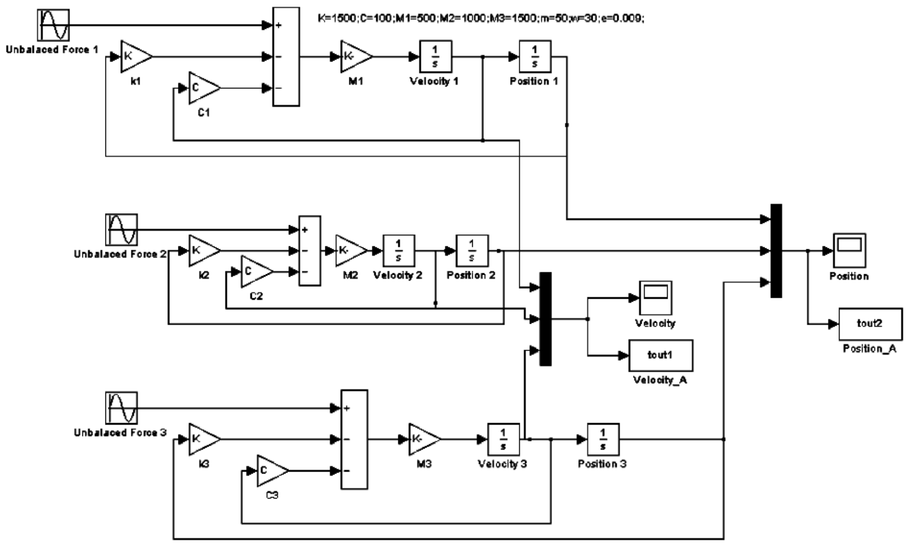

Figure 4. Scheme of equation that is evaluated.

In the previous scheme that is seen is the simulation of the next equation:

$$M\ddot{x} + c\dot{x} + kx = me\omega^2 \operatorname{sen} \omega t \tag{5}$$

In this scheme, what is done is to select functional blocks of a specific program of simulation, where each one performs a mathematical operations (sum, multiplication, logarithms, integrals…) represents a constant, a variable of the problem… what you do with these blocks is to fill them properly and unite them to build the desired equation.

But in this case we have done three simulations where are varying in each mass *"M"* which represents the mass of the static parts of the machine under study, in the first case is given the value of 500 kg. in the second of 1000 kg. and in the third 1500 kg. The other parameters are the same for all cases, friction constant *"c"*, stiffness constant *"k"*, the unbalanced mass *"m"* that generated the vibration…

RESULTS

The results of three simulations are the next figures:

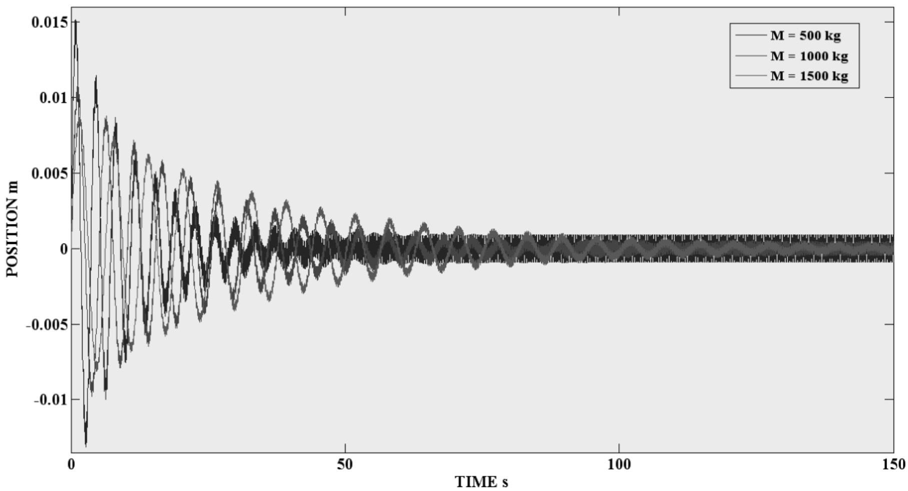

Figure 5. Results of a simulation, position versus time.

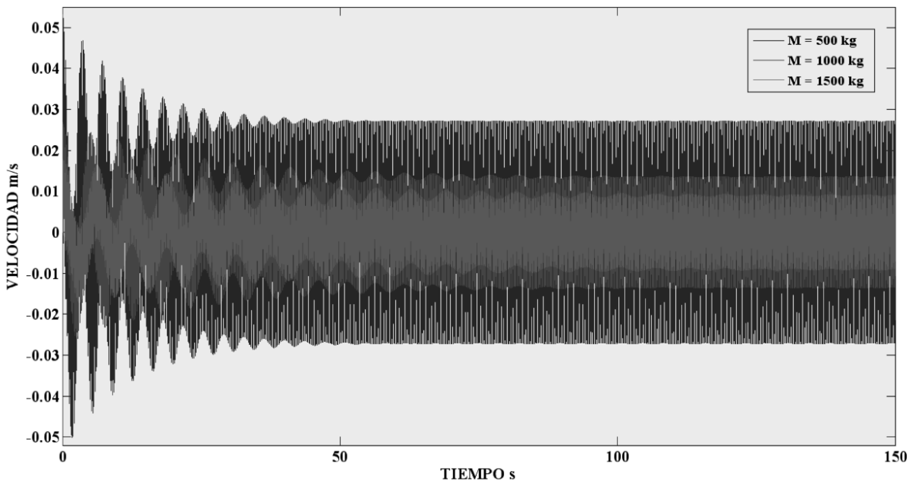

Figure 6. Results of a simulation, velocity versus time.

The figures 5 and 6 show the position and the velocity of vibration for one of each example simulated above. In this figures we can see the transient state and the steady state; in addition we can see how change the results when is changed the mass of static parts of the machine *"M"*.

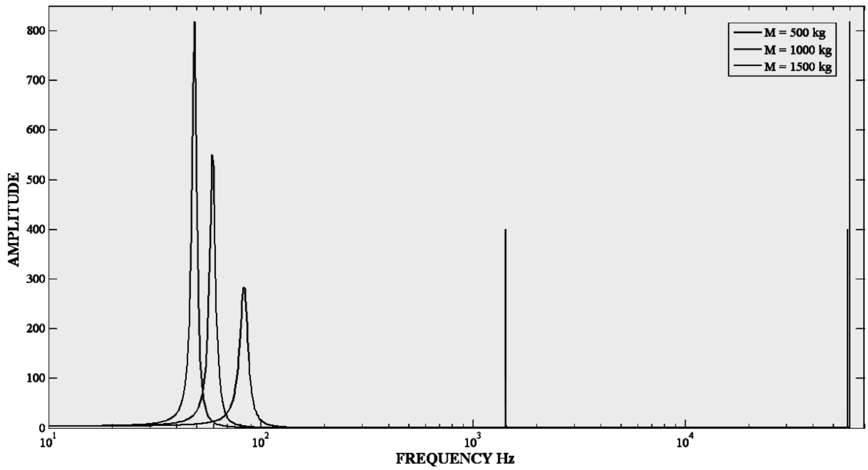

Figure 7. Results of a simulation, Spectrum of the systems.

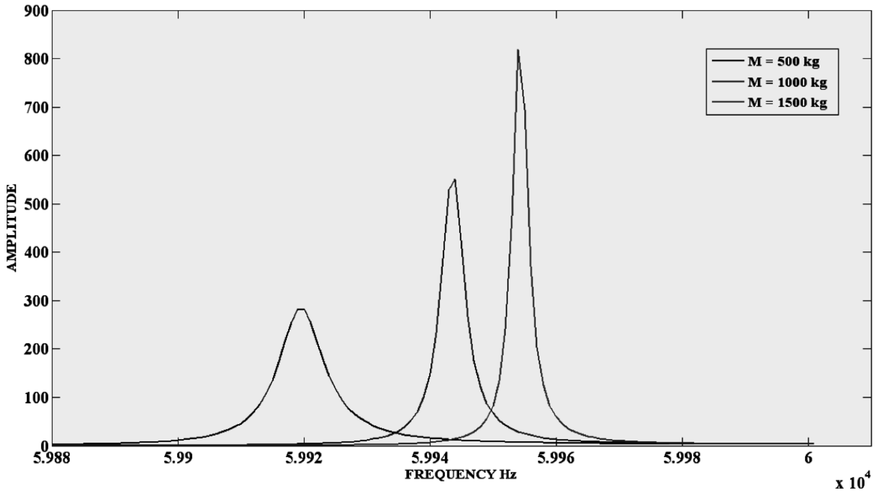

Figure 8. Results of a simulation, detail of a spectrum of the systems.

The figures 7 and 8 show the spectrum of vibration for one of each example simulated above. In this figures we can see the frequencies of the vibration; the frequencies between 30 and 100 Hz and over 60.000 Hz are the natural frequencies of vibration of each example and the frequencies over 1.400 Hz is the frequency of the excitation of unbalanced mass *"m"*.

From the above figures we can obtain a set of conclusion that would be difficult to obtain if we would have analyzed the equation mathematically and the response for each of the examples that we have assumed.

CONCLUSIONS

— The amplitude and the velocity (also the acceleration, as is not represented) of the vibration decrease when the mass of the machine *"M"* increase.

— The frequency of vibration on transient state is different for each instance, because of the variation of natural frequency "ω_n" by varying the total mass of the machine and not its stiffness.

— At steady state the only difference observed is the amplitude and the velocity of the vibration, but the frequency is the same for the three instances.

— By increasing the static mass increases the amplitude in the harmonics but decreases in the fundamental frequencies; and also the opposite happens; so the designer can modify the parameters of the vibration.

— The mass of the static parts does not have influence in the frequency during steady state, but it is fundamental on transitional arrangements.

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