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Application of QFT Order Reduction Methodology in the Control of an Autonomous Marine Vehicle

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ABSTRACT

Quantitative Feedback Theory (QFT) is a control methodology that allows you to work with mathematical models of high order, although it gives rise to compensators (controllers and/or prefilters) with order also high. The implementation of such controllers generates problems, even in some occasions unworkable. So, it's necessary trying to reduce the order of the model, to get low order controllers; or treating with the high order plant and reducing only the solution controllers. We are applying a QFT order reduction methodology for the course-changing control of a marine vehicle. The results are compared with those obtained with a classical QFT controller and with a PID controller.

1. Introduction

Some control methodologies, as Quantitative Feedback Theory (QFT), allow you to work with mathematical models of high order, but this means producing compensators (controllers G and prefilters F) with order also high. The implementation of these compensators is complicated, even in some occasions impossible. There are two solutions for this problem:

- Reducing the order of the original plant P before designing, getting lower order controllers.
- Designing the compensators with the high order original plant and, afterwards, trying to reduce these ones.

Obtained reduced functions will be considered valid always when they show similar behavior to the original ones. So, reduced plants will fulfill the design specifications in the same way than the original plant and, reduced controllers must maintain the same degree of stability and control than the original compensator.

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Suppose a transfer function (TF) given in the NUM/DEN polynomial form,

$$\frac{NUM(s)}{DEN(s)} = \frac{b_0 s^m + \cdots + b_m}{a_0 s^v + \cdots + a_n} \tag{1}$$

The objective is reducing its order, actually, reducing the number of poles and zeros, so that, the function (plant P_r or compensator G_r) will have got the most similar form as possible to the original, remaining its dynamics behavior. On the other hand, when the final function is a controller, it must continue maintaining the stability of the system. To get both things, the reduced function should fulfill next:

- $m \leq n$. More poles than zeros
- No right half plane (RHP) poles or zeros.
- No zero poles.
- The open loop reduced function ($L_r = PG_r$ with reduced controller or, $L_r = P_rG$ with reduced plant) does not cross the 2nd quadrant of Nichols chart (NC) or if it does, it fulfills at least conditional stability (Yaniv, 1999).

Reduction may be complicated on the original plant when the control problem contains a high degree of uncertainty, since the reduction process must apply over each one of the different model scenarios, which it may indeed result impracticable.

The procedure for rationalization of transfer functions to reduce them is described in (Joglar and Aranda, 2014). It can be applied both on plants and compensators through a matlab program (”RACW.M”).

We are applying this order reduction methodology for the course-changing QFT control of a marine vehicle. In particular, we start using the ship model described in (Rueda et al., 2001): the R.O.V. Zeefakkel, simplified with the first order Nomoto model. This control problem is solutioning with QFT and, afterwards, the original controller is reduced with the TF reduction order method presented here. The reduced results are compared with those obtained with the classical QFT solution developed and with the PID controller obtained by means of genetic algorithms proposed by (Rueda et al., 2001).

2. QFT Order Reduction Methodology

QFT methodology allows working with high-order plants, but usually this involves design of controllers and prefilters with also high order. In the process of shaping the open-loop TF or prefilters through the QFT IDE, you add typical design elements as, real or complex pole, real or complex zero, and others, to fulfill specifications. This makes the order resultant TF (controller or prefilter) grows, as much as more elements you add.

QFT order reduction methodology performs a reduced TF in numerator/denominator polynomial format with behavior as close as possible to the original TF, usually given in complex form within a certain frequency range w.

Talking about similar behavior between two TFs in two different formats, complex versus polynomial, means that both should have a similar appearance in the NC, offering magnitudes and phases for same frequencies as close as possible. So, we ensure that both TFs will fulfill design specifications similarly.

RACW.M is a function in Matlab which performs a RAtionalization process as it is described by (Horowitz, 1992). It works with complex input according to the supplied frequency vector w and offers as output the corresponding transfer function (numer/denom). It allows users to decide the order of the output functions, selecting the number of poles and zeros they may contain. The order of the output TF we have introduced will determine the difference in magnitude and phase, for each frequency within the range used, between the output and the input. So, selecting different output orders and observing the differences in magnitude and phase we can achieve reducing the order of the function to a value where these differences are not too high, what implies generally a similar behavior between input and output TFs in the NC.

The RACW.M program is formally described as follows,

```
function[numer,denom,error]=racw(c,w,n,m)    (2)
```

Where the input arguments of the function are,

c vector with size [N,1]; it's the complex input TF.

w vector with size [N,1]; it's the work frequency range.

n, m number of poles and zeros, respectively, for the output reduced TF in the form 'numer/denom'.

Between the output reduced FT and input vector c(s) there is the following relationship,

$$\frac{numer}{denom} = \frac{b_0 + b_1 s + \cdots + b_n s^n}{a_0 + a_1 s + \cdots + a_m s^m} = c(s), \quad \text{with } s = jw \quad (3)$$

Which can be rearranged as,

$$[a_0+a_1 s+\cdots+a_{m-1} s^{m-1}]-\frac{1}{c}[b_0+b_1 s+\cdots+b_n s^n] = -a_m s^m \quad (4)$$

From this equation, we can develop N more equations replacing the input function of N complex numbers c(s), one for each frequency w. Applying the rationalization process described by (Horowitz, 1992), it is obtained values for the m coefficients a_i and n coefficients b_i.

Now, if we perform a polynomial evaluation of the TF with the coefficients a_i and b_i obtained, comparing in magnitude and in phase with the value of the original complex TF $c(s)$, errors in magnitude and phase of the process are leading.

3. Course-Changing Control Problem of an Autonomous Marine Vehicle

We are using QFT as a robust control methodology in the design of the compensators necessary for the system with uncertainties described in (Rueda et al., 2001). The model of the ship, a vessel of 45m in length the R.O.V. Zeefakkel, may be approximated by the next equation (plant P(s)), which relates the heading angle Ψ as input command, with the rudder angle δ as output command from the controller to the steering gear (Fossen and Paulsen, 1992),

$$P(s) = \frac{\Psi}{\delta}(s) = \frac{K}{s(1+sT)} \quad (5)$$

At a speed of 10knots, the uncertainty of the model is given by the following variations in the K and T parameters,

$$K \in [0.21, 0.5]; \quad T \in [29.5, 5.31] \quad (6)$$

It will be a two-degrees of freedom LTI control system performed with a controller G and a prefilter F, in order to reduce the output variations of the parametrically uncertain plant P.

In order to achieve robust stability and robust tracking, the system must fulfill the following specifications:

- Stability margins: phase margin at least 45^o and gain margin bigger than 2dB, formally described as,

$$\left|\frac{PG}{1+PG}\right| \leq \rho, \quad \text{with robust margin weight, } \rho = 1.2 \quad (7)$$

- Tracking: expressed by the lower T_L and upper T_U tracking bounds respect to the closed loop transfer function T_R,

$$T_L(s) \leq T_R(s) \leq T_U(s) \quad (8)$$

With $s = jw$ and for $\omega \leq 0.4 rad/s$, being,

$$T_R = \left|F \frac{PG}{1+PG}\right| \quad (9)$$

$$T_L = \frac{269.5 * 10^{-6}}{q(s)} \quad (10)$$

$$T_U = \frac{195 * 10^{-4} s + 49 * 10^{-4}}{s^2 + 122 * 10^{-3} s + 49 * 10^{-4}} \quad (11)$$

for $q(s) = s^3 + 181 * 10^{-3} s^2 + 118.3 * 10^{-4} s + 269.5 * 10^{-6}$

For the control design, it has been chosen the nominal plant P0 given by,

$$P_0(s) = \frac{0.21}{s(29.5s+1)} \quad (12)$$

And we have established the following set of frequencies,

$$w = \{0.03, 0.07, 0.1, 0.2, 0.4, 1, 1.2\} \quad (13)$$

Using the interaction design environment (IDE) of the QFT Toolbox in Matlab designed by (Borghesani et al., 1995) the boundary of plant templates is computed for each frequency of the work set: 28 plants at 7 frequencies. Observe the plant templates computed in the Figure1, where the nominal plant is marked and occupies, at each frequency, the lower magnitude position in the NC.

After computing bounds independently, robust margins and robust tracking, they are grouped and the intersect bounds are processed. Now, it can be started the design process. Adjusting the nominal open-loop transfer function L0=P0G in an adequate way, we ensure no bounds are violated and specifications are fulfilled.

The controller obtained in the shaping process of L_0 is (3, 3) order, i.e. with 3 zeros and 3 poles, shown in the Figure 2 and formally described as,

$$G_{(3,3} = \frac{1.195s^3 + 4.5386s^2 + 1.4856s + 0.0365}{s^3 + 1.6853s^2 + 2.0323s + 0.1873} \quad (14)$$

Adjusting the prefilter F with the IDE of the QFT Toolbox in Matlab, it is obtained,

$$F(0,1) = \frac{0.04915}{s + 0.004915} \quad (15)$$

The frequencial analysis offers the results shown in the Figure3 and Figure4, fulfilling margins and tracking specifications.

At last, in the Figure5 we can see the tracking of the control system for a course-change of 10^o in the time domain. The solid lines represent the different scenarios of the plant due to its uncertainty. The lower tracking bound is violated in a minimum part.

4. Application of QFT Order Reduction Methodology

Applying the reduction procedure described above, we get a reduced TF for $G(3,3)$, the controller $G_r(1,1)$, whose graphical representation in the IDE of the Toolbox QFT in Matlab has the appearance shown in Figure 6.

The results obtained from the comparative analysis between the Figure 2 and Figure 6 are:

- Same behavior in both functions of magnitude and phase, in the frequency range ($0.03 rad/s - 1.2 rad/s$). It defines the same robustness of the two TFs in the frequencial domain, fulfilling every specification.
- Correct gain margin (GM) and phase margin (PM). They fulfill the specification for robust stability (RS), expressed as,

$$|GM| \approx 99dB \geq 2dB = GM_{RS} \quad (16)$$

$$|PM| \approx 66^o \geq 45^o = PM_{RS} \quad (17)$$

- One crossover frequency, similar in both cases.
- Reduced TF fulfills tracking specifications in a similar way as the original TF.

The reduced TF G_r has the order (1,1), ie, one zero and one pole and, it is given by the following expression:

$$G_r(1,1) = \frac{2.9409s + 0.197}{s + 0.611} \quad (18)$$

The magnitude and phase errors for the reduced TF $G_r(1,1)$ respect to the original TF $G(3,3)$ in absolute terms and, for the predetermined frequency range w, are described in the Table1 below and afterwards, graphically in Figure7.

The magnitude and phase errors obtained for reduced function $G_r(1,1)$ compared with the original TF, within the whole frequency range w, are small enough to consider it appropriate, in this regard.

The frequencial analysis offers the results shown in the Figure8 and Figure9, fulfilling margins and tracking specifications. Observe results in tracking specifications with $G_r(1,1)$ are better than with the original controller $G(3,3)$.

Finally, Figure10 shows the tracking of the control system for a course-change of 10^o in the time domain using $G_r(1,1)$. Results are much better than those with the original controller $G(3,3)$.

Now, we are comparing the reduced results generated with the $G_r(1,1)$ controller with those obtained with the PID controller produced by means of genetic algorithms proposed by (Rueda et al., 2001). The controller G_{PID} has the order (2,2), ie, two zeros and two poles and, it is given by the following expression:

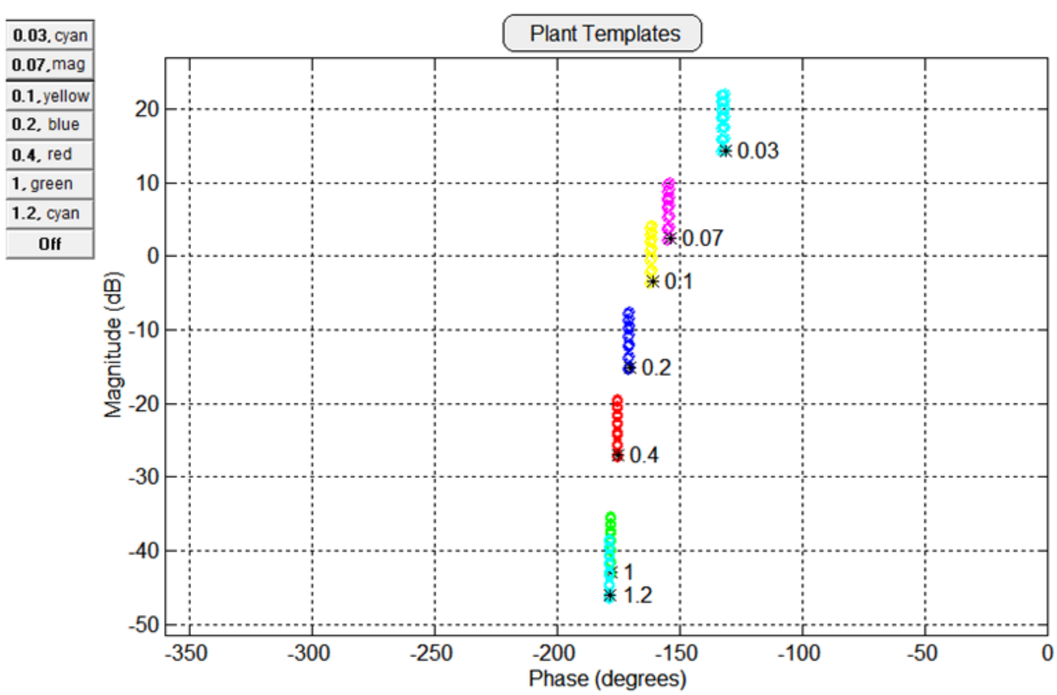
$$G_{PID} = \frac{15523.7s^2 + 448.37s + 0.1}{44737s^2 + 4473.7s} \quad (19)$$

Figure 1: Characteristics of the reduced TF $G_r(1, 1)$ respect to the original function $G(3, 3)$ in the frequency range w

w ***rad/s***	***C(in)***	***C(out)***	***Mag Error c(in)-c(out)***	***Phase Error Phase[c(in)]-Phase[c(out)]***
0.03	0.2285+0.1648j	0.3287+0.1283j	0.1066	14.4737
0.07	0.3305+0.3165j	0.3563+0.2961j	0.0329	4.0315
0.1	0.4044+0.3848j	0.3907+0.4174j	0.0354	-3.3146
0.2	0.5613+0.5329j	0.5758+0.7742j	0.2417	-9.8473
0.4	0.7831+0.8348j	1.108+1.2j	0.4888	-0.4532
1	2.1284+1.2727j	2.2291+1.165j	0.1475	3.2862
1.20	2.6001+0.9512j	2.4018+1.0587j	0.2256	-3.6936

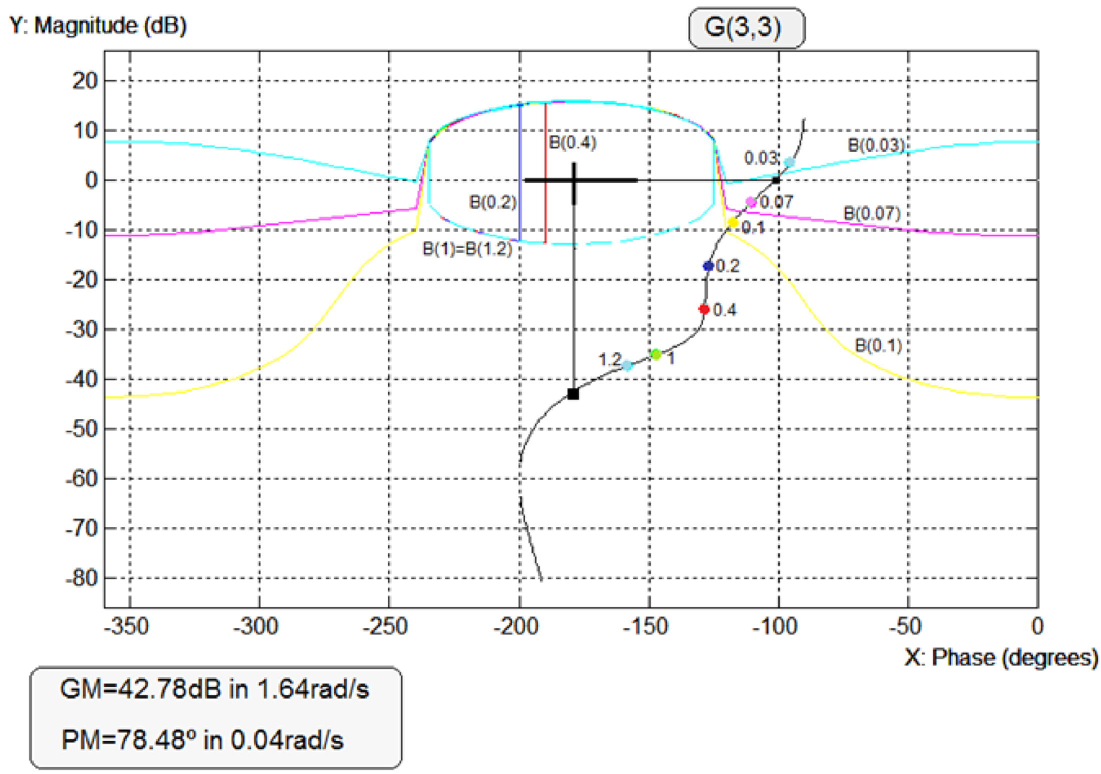
Source: Authors

Figure 1. Plant Templates in the Nichols Chart



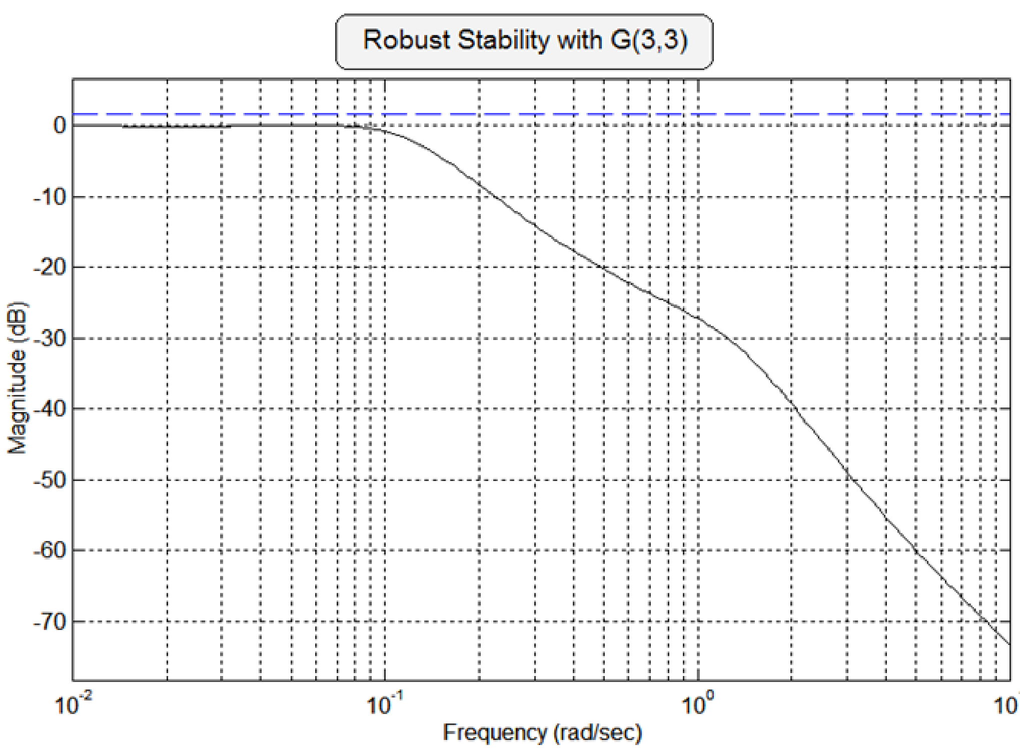
Source: Authors

Figure 2. Shaping of L_0, obtaining $G_{(3,3)}$ controller



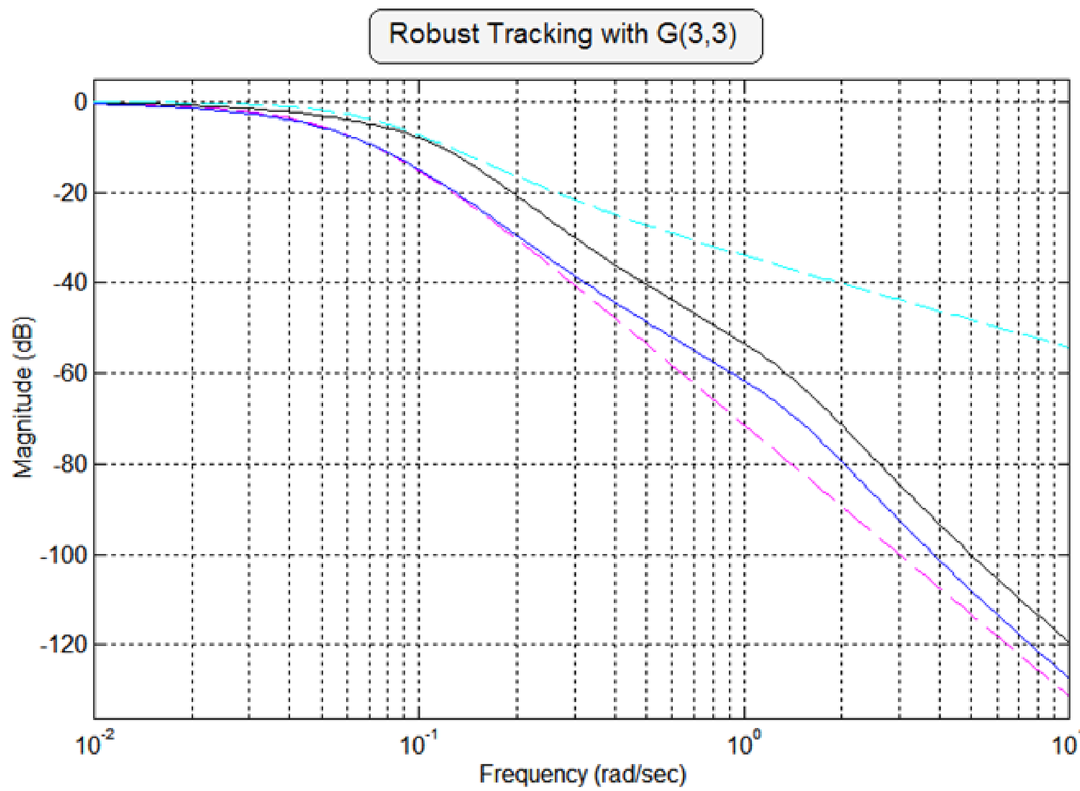
Source: Authors

Figure 3. Stability margins results in the frequency domain using $G_{(3,3)}$



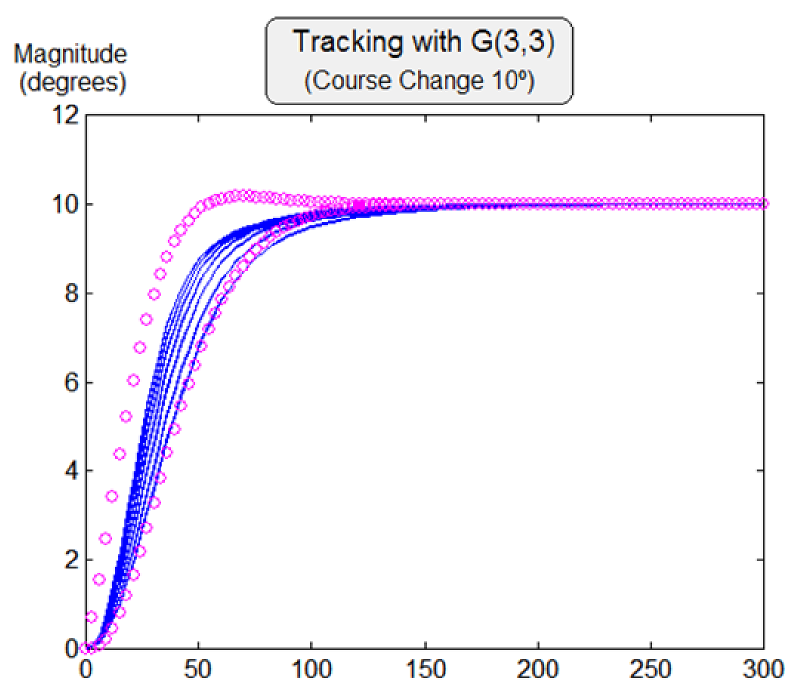
Source: Authors

Figure 4. Tracking results in the frequency domain using $G_{(3,3)}$



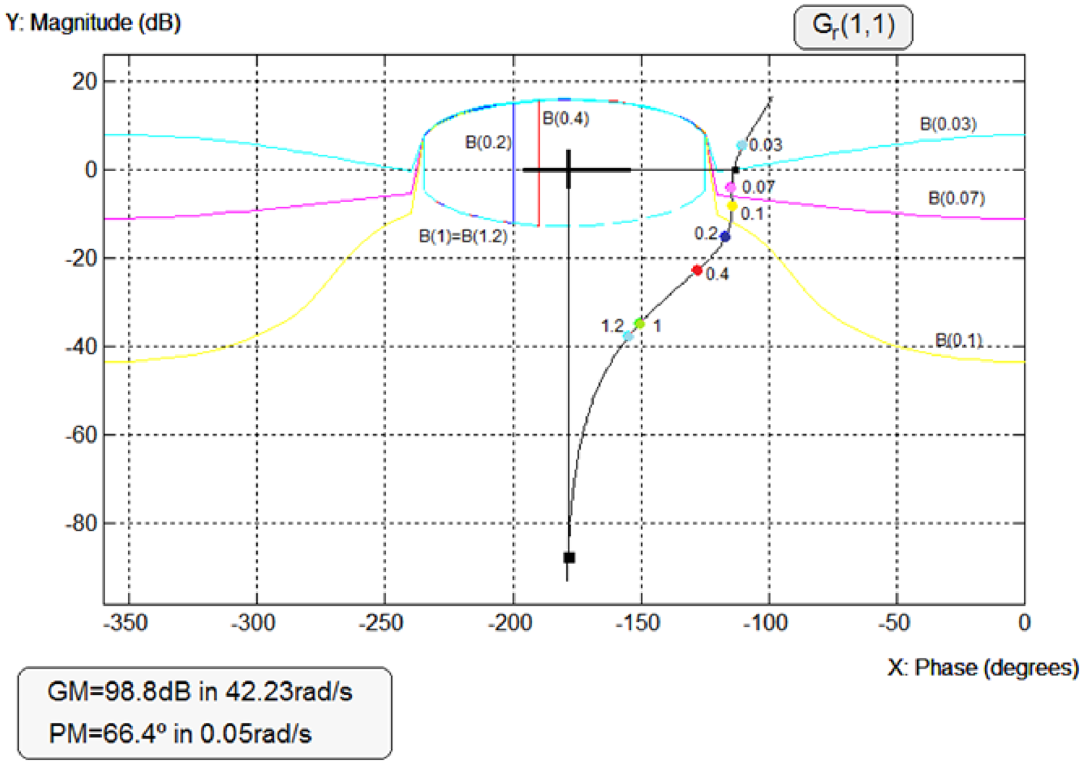
Source: Authors

Figure 5. Tracking results in the time domain using $G_{(3,3)}$. Course-change of 10^o



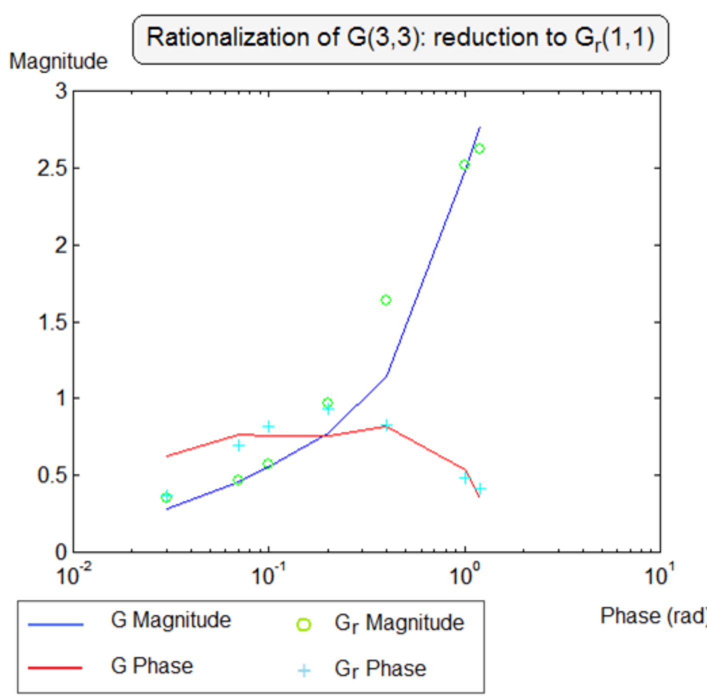
Source: Authors

Figure 6. the reduced open loop function with $G_r(1, 1)$ controller



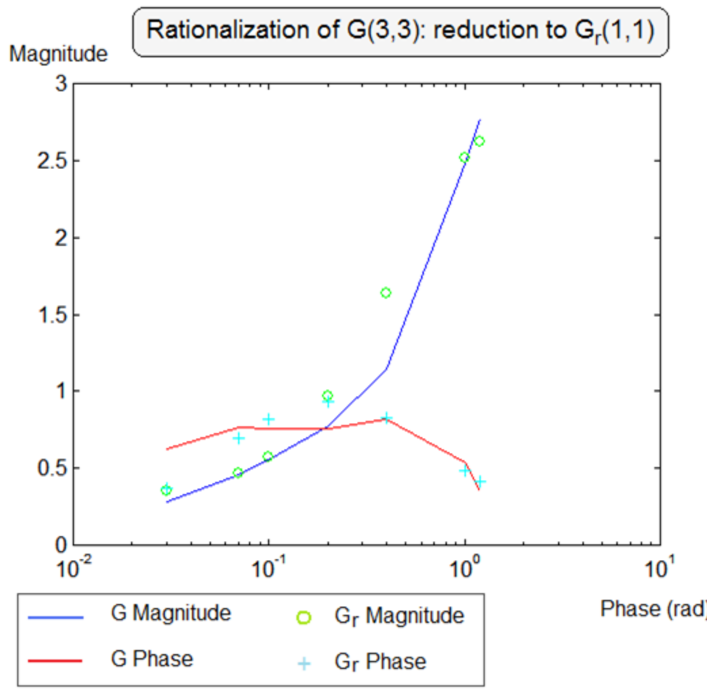
Source: Authors

Figura 7. Graphic features of $G_r(1, 1)$ compared with $G(3, 3)$, applying `RACW(G,w,1,1)`



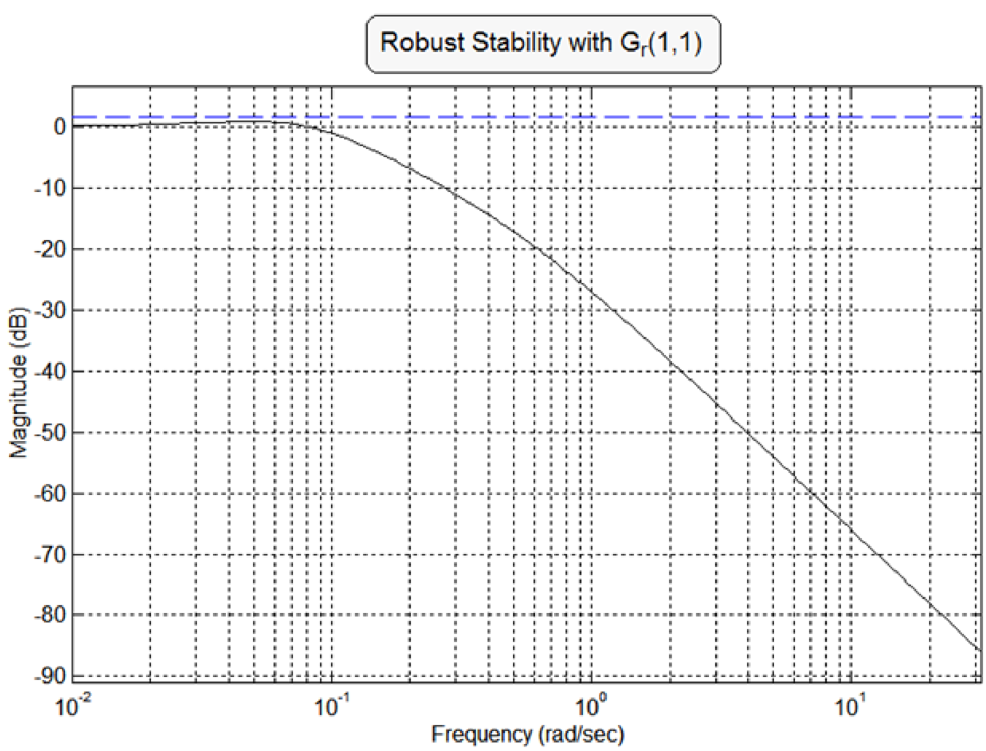
Source: Authors

Figura 7. Graphic features of $G_r(1,1)$ compared with $G(3,3)$, applying `RACW(G,w,1,1)`



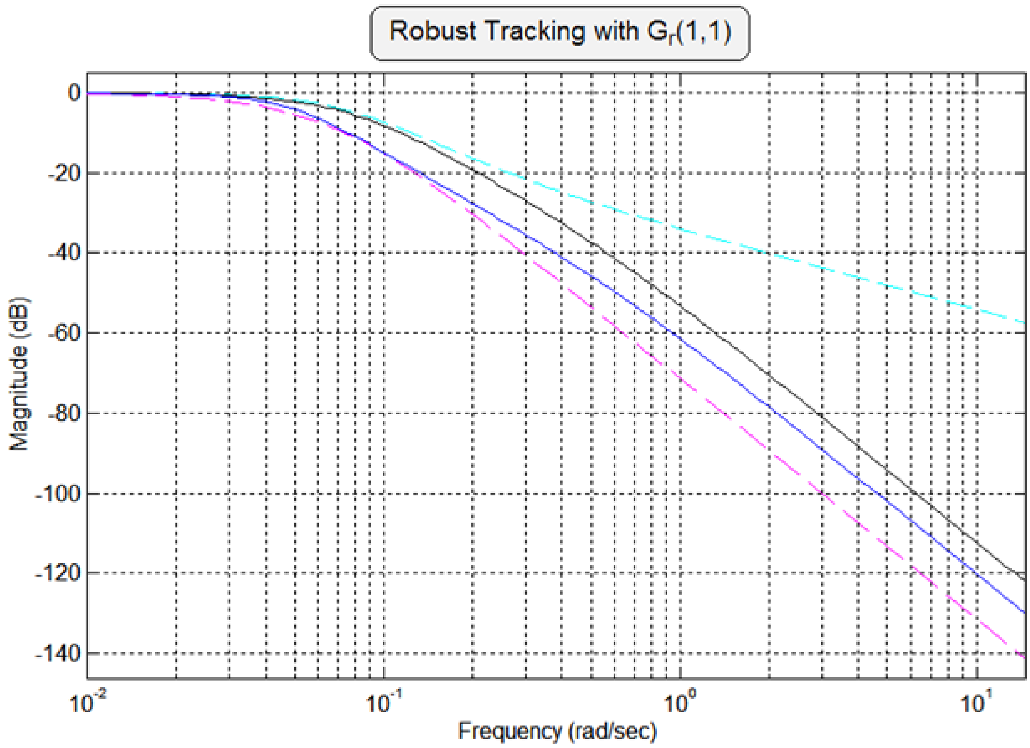
Source: Authors

Figura 8. Stability margins results in the frequency domain using $G_r(1,1)$



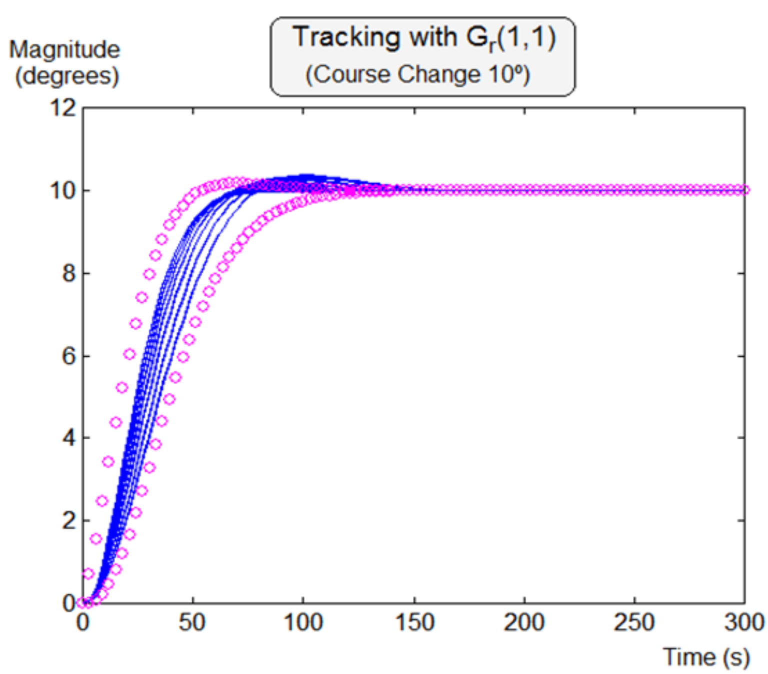
Source: Authors

Figura 9. results in the frequency domain using $G_r(1, 1)$



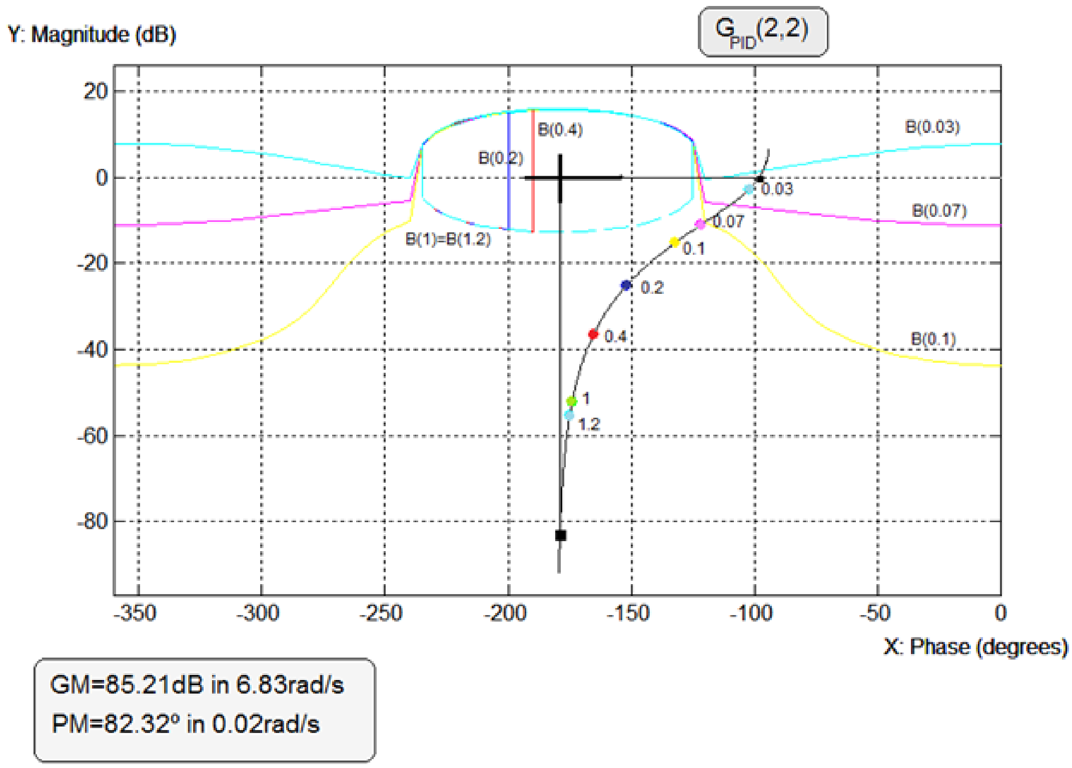
Source: Authors

Figura 10. Tracking results in the time domain using Gr(1,1). Course-change of 10^o



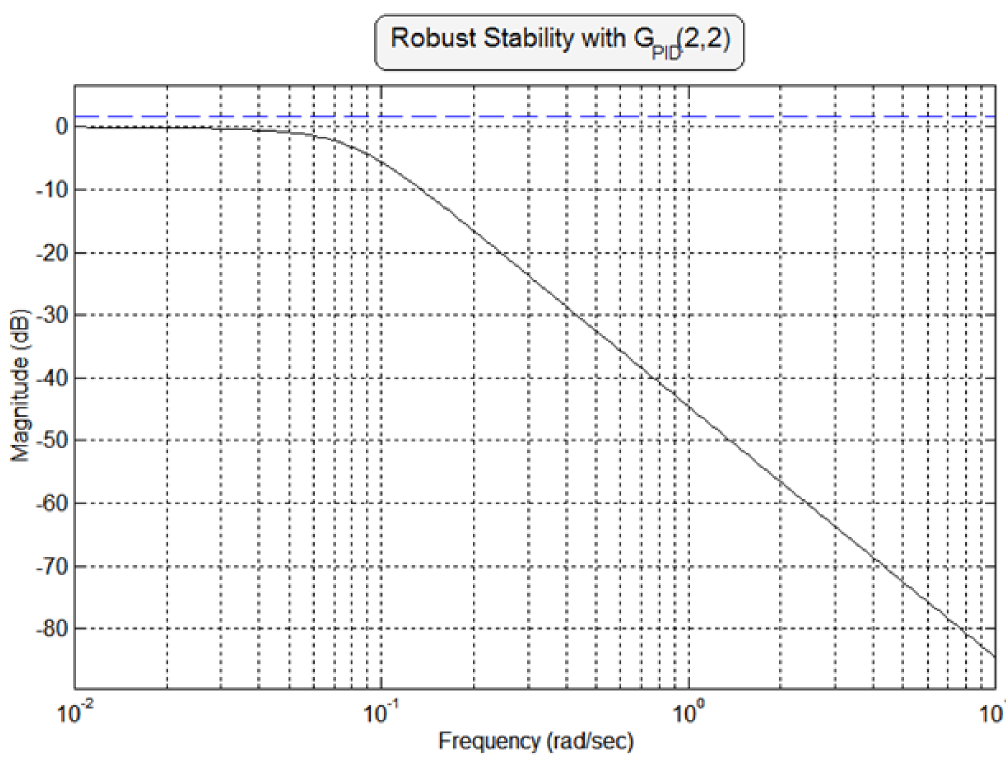
Source: Authors

Figura 11. Setting the open loop function with $G_{PID}(2,2)$ controller



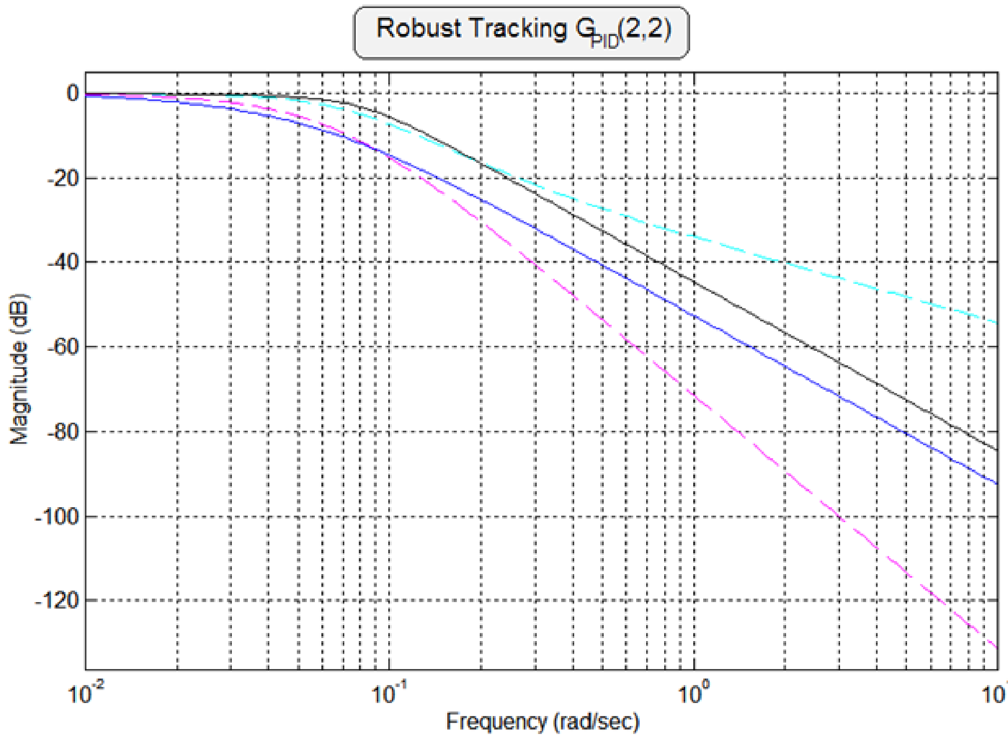
Source: Authors

Figura 12. Stability margins results in the frequency domain using $G_{PID}(2,2)$



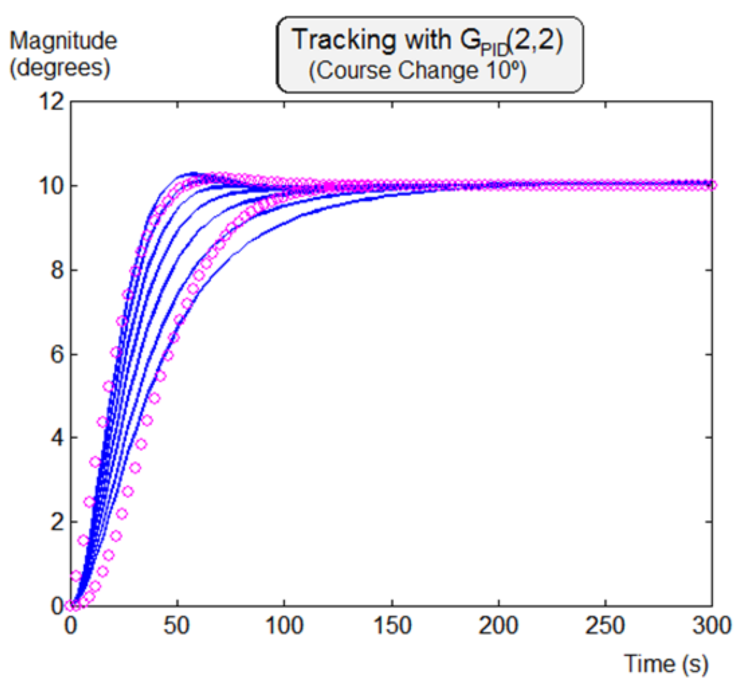
Source: Authors

Figura 13. Tracking results in the frequency domain using $G_{PID}(2,2)$



Source: Authors

Figura 14. Tracking results in the time domain using $G_{PID}(2,2)$. Course-change of 10^o



Source: Authors

For this, using the shaping procedure of the open loop function with the $G_{PID}(2,2)$ controller, as shown in Figure11, we can compare the results with those in the analysis of the $G_r(1,1)$ controller application. So,

- Different and incorrect behavior, in the frequency range $(0.03rad/s - 0.1rad/s)$. The open loop function with $G_{PID}(2,2)$ controller doesn?t fulfill every specification in the frequencial domain.
- Correct gain margin (GM) and phase margin (PM). They get fulfilling the specification for robust stability (RS), expressed as,

$$\left|GM\right| \approx 85dB \geq 2db = GM_{RS} \tag{20}$$

$$\left|PM\right| \approx 82^o \geq 45^o = PM_{RS} \tag{21}$$

- One crossover frequency, similar in both cases.
- It doesn?t fulfill tracking specifications, as shown in Figure13.

The frequencial analysis offers the results shown in the Figure 12 and Figure 13, fulfilling margins specifications but not tracking specifications.

Finally, Figure 14 shows the tracking of the control system for a course-change of 10^o in the time domain using $G_{PID}(2,2)$. Observe, results are worse than those with the original controller $G(3,3)$ and much worse than results offer with the reduced controller $G_r(1,1)$.

5. Conclusions

In a control problem it is often interested the lowest order solutions as possible, in technologically and/or economically practical terms. Always there must be a compromise solution between effectiveness and simplified controller order. There are two ways to get it: trying to reduce the order of the original plant to obtain directly low-order controllers; or reducing the solution controllers of high order, obtained from the original plant.

Anyway, the used order reduction procedure must ensure final reduced functions have a similar behavior than the original ones. So, in the first case, the reduced plants will fulfill the design specifications in the same way as the original plant and, in the second case, the reduced controllers must maintain stability and control with the same degree as the original compensator.

The goodness of the reduction methodology of any polynomial transfer function has been proved with the example of an autonomous course-changing marine vehicle. First, the control problem has been solved applying QFT. Then, it has been used the reduction procedure over the previous high order controller to obtain a simplified controller, which offers a similar dynamics behavior of the control system. At last, it has been performed a comparative between the obtained results using a PID controller and both former cases.

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