



Numerical failure Assessment of a Double-hull Tanker Ship colliding with a Container Ship

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ABSTRACT

The risk of ship collisions has increased rapidly, particularly with high maritime traffic. Consequently, an extensive demand is required for environmental protection and safety at sea. Structural failure during a collision event must be considered in the structural design stage. This study aims to assess numerically the structural response of a double-hull tanker ship colliding with a striking container ship. A numerical model is performed, using nonlinear finite element analysis. To improve the numerical results, true stress-strain and failure criteria of material were used. Fine elements were used at impacted area to predict the local deformation accurately. Finally, a parametric study has been carried out to investigate the effects of the ship velocity, impact region, friction coefficient, and collision angle on the total dissipated energy of the impacted ship. The obtained results can contribute to improving the structural integrity of ship against external loading.

1. Introduction.

Based on reported maritime accident documents worldwide, ship collisions are the predominant event, representing more than 29% of maritime incidents (Hong, 2009), which was leading to extensive structural damage to ships, including hull, decks, and superstructures (Liu et al., 2021; Rhidian Thomas, 1991). In severe cases, the damage may lead to the sinking of one or both ships and potential environmental pollution. One of the

most tragic consequences of ship collisions is the injury of crew members and passengers.

Therefore, a series of research projects have been conducted to better understand the structural response of ships during collision events. Moreover, Maritime classification Societies have implemented new rules and guidelines to enhance the safety of ships against catastrophic failures caused by collision or grounding (Egge and Böckenhauer, 1991; Liu and Guedes Soares, 2023). Ship collisions are characterized by a high degree of uncertainty and an expected damage state. To accurately assess the structural failure of ships, collision variables such as ship velocity, impact location, and angle must be considered (Sormunen et al., 2015). According to literature, both analytical and numerical methods have been used to assess the structure of a ship subjected to dynamic loading, where the numerical models are still important in the design phase and can be extremely useful for ship collision accident, aid in decision-making on which measures should be applied until repairs are carried out (Liu et al., 2021a). The non-linear finite element method is widely used for a complex phenomena such as dynamic collision, containing a high non-linear structural deformations (Pineau, 2022) and stress analysis of structural discontinuities (Hoque and Islam, 2023). In recent decades, extensive researches have been

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mainly focused on the assessment of ship structure against dynamic loading. Among these studies, (Wang et al., 2000) investigated the influence of penetration's location and bow radius on the behavior of a double hull. (Wiśniewski and Kołakowski, 2003) studied the effect the impact velocity, collision angle, and structural arrangement during a collision response of ships. (Ozguc et al., 2006) used numerical simulation and experimental to assess the structural integrity of single and double skin bulk carriers subjected to collision damage. (Ehlers et al., 2008) have studied the sensitivity of failure criteria for collision simulations and grounding. Recently (Ehlers, 2010) investigated the influence of the material relation on the accuracy of reliability of the finite element model. (Zhou, 2013) used theoretical and numerical methods to predict the energy dissipation of soft spheres during normal impacts. (Haris and Amdahl, 2013) considered the interaction between bow and side deformations to analyze the collision damage. (Bae et al., 2016) evaluated the influence impact velocity, ship size, and structural characteristics. To enhance numerical simulations (Calle et al., 2017) considered the mechanical properties of the materials and failure criteria. Experimental analysis has been used in ship collision and grounding. (Liu et al., 2018) presented a review of experiments and calculation procedures for the resistances of ship structural components subjected to impact loadings. (Zhang et al., 2019) estimated the energy absorption and damage extent for severe ship collision damages and validated with experimental data. (Liu et al., 2021) analyzed several collision scenarios, based on statistical data of striking ship dimensions, velocities, collision angles and locations, as well as seabed shapes and sizes, grounding depth and location. (Zhang et al., 2023) conducted an experimental and numerical simulation studies on a scaled ship side-shell quasi-statically punched at the mid-span by a raked bow indenter.

The present study evaluates the structural response of a double-hull tanker ship colliding with the rigid bow of a container ship. Explicit nonlinear finite element analysis (commercial software Abaqus/ (Dassault Systèmes, 2017) was employed to simulate the collision scenarios. Energy dissipation, penetration and damage extent were also computed. To ensure the reliability of numerical results, plastic deformation behavior was performed, and a refined mesh was used in the impacted region to account the nonlinearity of the geometry and materials. Finally, a parametric study has been conducted to emphasize the influence of several parameters, including the ship's velocity, impacted region, friction coefficient, and collision angle on the structural response of impacted ship. These results could be used to enhance the structural arrangement design of tanker ships and reduce the environmental pollution.

2. Numerical Model preparation for Ship-Ship Collisions.

2.1. Calibration and validation of ductile material model.

Large structural deformations are usually occurred in ship collision. For this, a suitable choice of the plastic material behavior is strongly required to accurately represent the state of stress and strain until failure (Liu and Guedes Soares, 2023).

Material calibration based on the engineering tensile test is important to enhance the numerical results and to obtain a reliable finite element model (MacLean, 2012). In addition, stress triaxiality is highly recommended (Ganjani and Homayounfar, 2021; Sajid and Kiran, 2018). Fig. 1. shows a comparison between the numerical tensile model and true stress-strain experiment. It can be observed that the two curves almost coincide, and good results are achieved. However, a slight difference was observed in the plateau region. The plastic strain is given as follows:

$$S_t = \frac{F}{A} = S_e(1 + e_e) \quad (1)$$

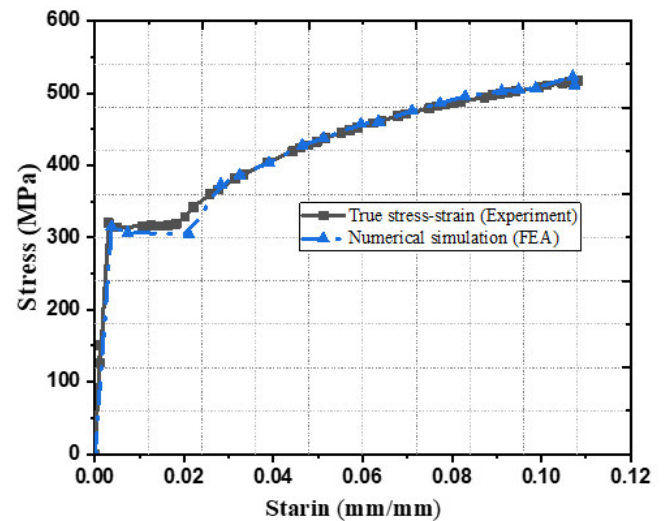
$$e_t = \ln(1 + e_e) \quad (2)$$

$$e_p = e_t - e_e = e_t - \frac{S_t}{E} \quad (3)$$

where:

S_t	True stress;	S_e	Engineering stress;
e_t	True strain;	e_p	Plastic strain;
e_e	Engineering strain;	E	Young's Modulus;

Figure 1: True Stress - Strain tensile curve for steel grade A36.

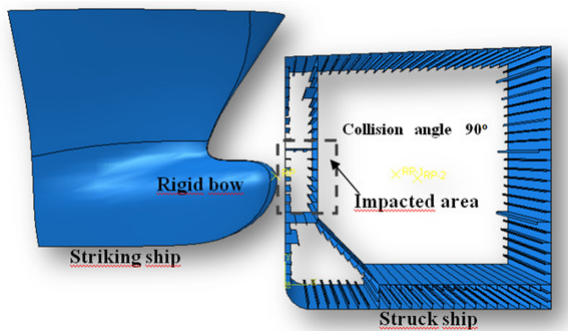


Source: Authors.

A three-dimensional numerical model is performed to assess the structural response of the double-hull tanker ship collided by a rigid bulbous bow. This model is consisted of striking ship, represented by a full bow of container ship which has two parts (upper, and lower), and a struck ship represented by the double-hull tanker ship. The scantlings and detailed structural dimensions of the tanker's amidship section are depicted in Fig. 2 and 3. The full bulbous bow geometry was imported from Maxsurf software. Dimensional characteristics of the both struck and striking ships are given in Table 1. where the displacement of the container ship is 111563.23 t with 7.239 m of draft. The study presented here is based on a right angled collision scenario (90°), without taking into account the movements

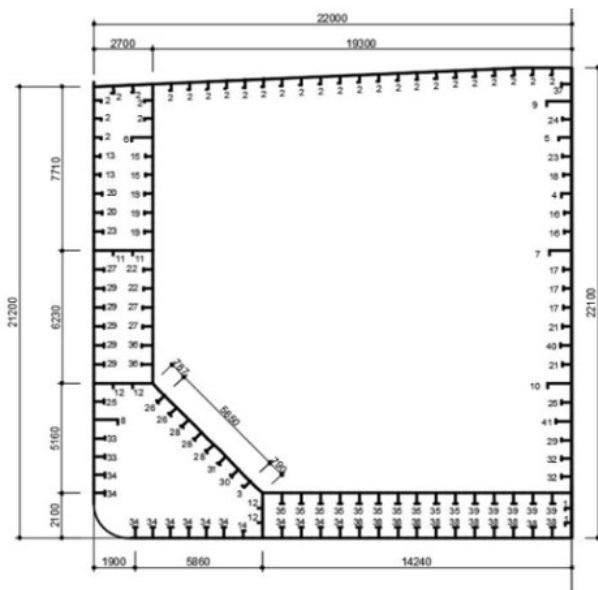
of the colliding ship and their interaction with the surrounding water (Kuznecovs et al., 2023). According to (Chen et al., 2022) the total absorbed energy by the rigid and deformable bow is relatively similar, hence, a bulbous bow is assumed to be rigid, where the only side structure of impacted ship will deform. The velocity of striking ship is 10 m/s, while the struck ship is assumed to be withstand. The numerical simulation is typically carried out using explicit dynamic analysis methods, considering the transient and nonlinear behavior during the impact loading.

Figure 2: Collision scenario details of the double-hull tanker ship with a rigid bulbous bow - Full numerical model of ship collision.



Source: Authors.

Figure 3: Collision scenario details of the double-hull tanker ship with a rigid bulbous bow - Amidships sectional area a double-hull tanker ship.



Source: Authors.

Table 1: Dimensional characteristics of a struck ship (Double hull oil tanker ship).

Dimensional characteristics		Striking ship - Double hull Oil tanker ship	Struck ship - Container ship
	Length over all (m)	234	116,214
	Breadth (m)	44	19,5
	Depth (m)	22.1	13,586
	Bottom (mm)	0.89	-
Frame space	Hull (mm)	0.89	-
	Deck (mm)	0.8909	-

Source: Latumahina et al., 2018.

In order to represent accurately the material behavior of ship's hull during the collision event, the failure criterion of maximum shear stress is used, which represents the initiation of a damage due to a zone of intense deformation by shearing (Amdahl, 2017). Material properties of the impacted ship are given in Table 2.

Table 2: Material Properties of the double hull oil tanker ship .

Parameters	Value
Density ($\text{kg} \cdot \text{m}^{-3}$)	7850
Elastic modulus (MPa)	210000
Poisson's ratio	0,30
Yield stress (MPa)	290

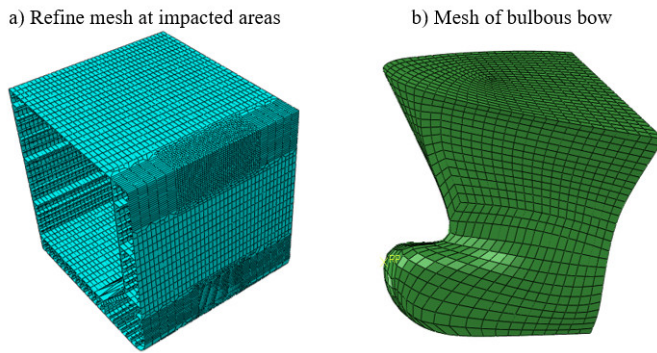
Source: Latumahina et al., 2018.

It should be noted that structural failure mainly occurs in the impacted area of the struck ship. Therefore, the accuracy of the simulation results was strongly linked to the appropriate boundary conditions and refined elements in the impacted area. Further, an appropriate mesh refinement should be used to capture all structural deformations resulting from the collision. (Liu et al., 2018) . Fig.4.a shows the mesh refinement at contact area of the outer shell side. A quadrilateral shell element (S4R element) is used in this simulation (Obisesan, 2017) . To better visualize the effect of the impact on the ship's hull, we released the degrees of freedom that corresponding to the impact direction. The impacted region subjected to fixed boundary conditions in the longitudinal ends.

2.2. Investigation of hull fracture.

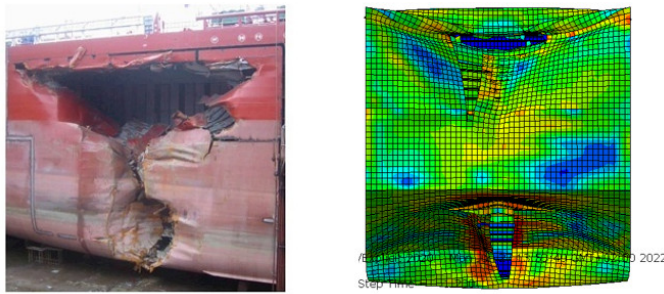
During the collision process, different parts of the ship structure were consecutively affected. The outer hull was the first part damaged by the collision force, and the decks situated between the two hulls were subjected to plastic deformation. Finally, the inner hull is in the final phase of structural deformation. It was observed that immense structural deformation occurred at the contact region where the stiffeners were completely ruptured. In addition, the two regions were fractured by the upper and lower parts of the rigid bow (Fig.5.).

Figure 4: Mesh density and finite element type of ship-ship collision.



Source: Authors.

Figure 5: Example of structural damage of outer side-hull ship.



Source: Zhang et al., 2019 / Authors.

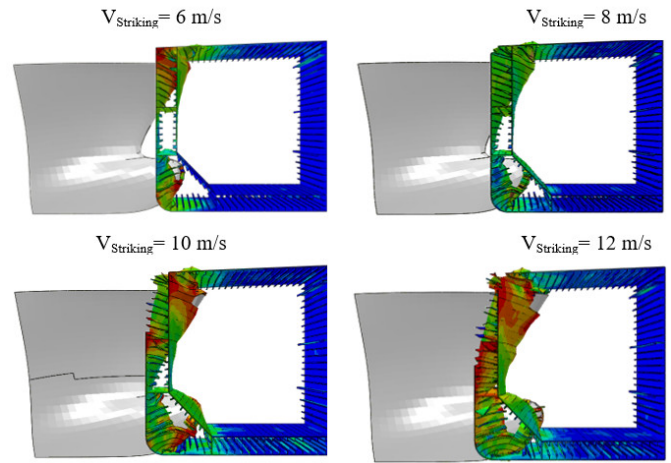
2.3. Parametric study.

2.3.1. Effect of a striking ship's velocity.

A parametric study was conducted to investigate the influence of several collision parameters, including the ship's velocity, impact location, friction coefficient, and collision angle, on the double-hull tanker. In this section, a range of striking ship velocities is proposed, varying from 6 to 12 m/s. Graphical results in Fig. 6. showed that the extent of damage and penetration depth increased when the striking velocity increased. The most significant penetration occurred at the highest velocity of 12 m/s, where the bow penetrated the outer and inner hulls of the struck ship. However, at ship velocities of 6 and 8 m/s, only the outer hull side fractured. Based on the above finding, higher velocities lead to more severe damage and deeper penetration.

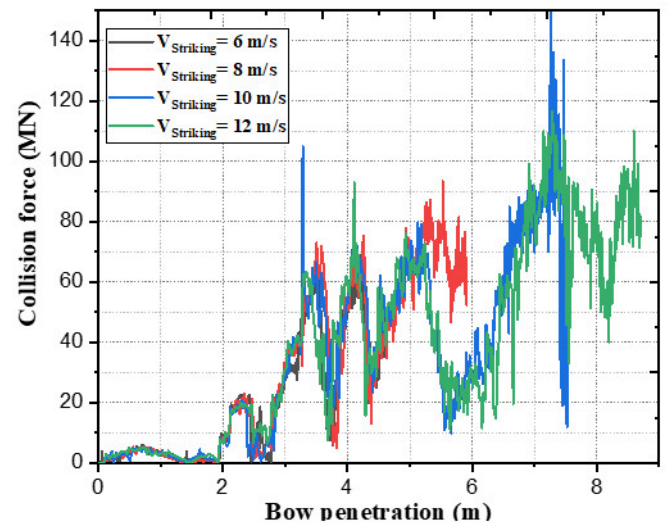
Fig. 7 shows the collision force history as a function of bow penetration during the collision process for different velocities. All curves follow a non-linear relationship, indicating a complex interaction between the impacted area and the rigid bow. Peaks and fluctuations were recorded in the range of 0.0 to 1.2s.

Figure 6: Collision interaction scenario at different velocity of the striking ship.



Source: Authors.

Figure 7: Collision forces versus bow penetration at different ship velocities.

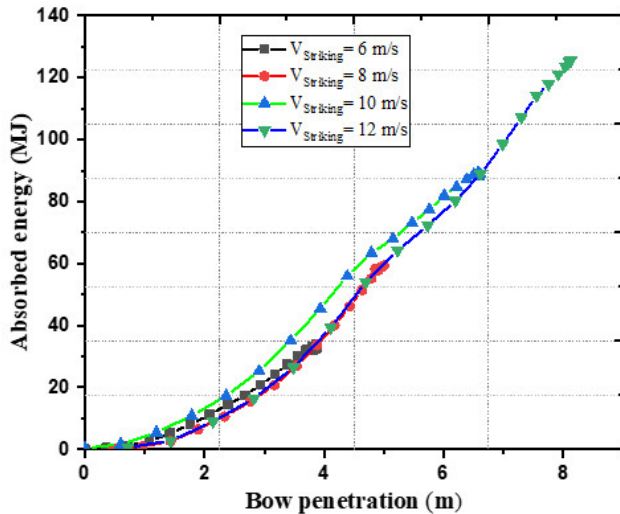


Source: Authors.

Each of these peak values denotes the critical point at which the bow penetrates structural parts. Two maximal peaks were observed. the first one is related to the fracture of the outer hull side and the second is accompanied by the fracture of the inner hull side. A strong decrease in the impact force was noted due to the total fracture of the impacted region. It is shown also a progressive penetration of the bow into the hull side of the ship, where the displacement increased steadily. This increase in displacement directly correlates with the collision force applied to the struck ship. The deepest penetration was observed at a velocity of 12 m/s. The maximum displacements reached by the velocities (6, 8 m/s, 10 m/s, and 12 mm/s) were (4.59m, 5.89m, 7.39 m, and 8.59 m) respectively. Fig. 8. illustrates the absorbed energy in terms of displacement. The total absorbed

energy by the struck ship at the velocity of 12 m/s is 125.58 MJ and is only 33.76 MJ at the velocity of 6m/s. Considering that the bow is rigid, the total kinetic energy of the striking ship was consumed to penetrate the side hull. More energy is consumed during a collision at a higher velocity.

Figure 8: Dissipated energy of the struck ship for different velocities.



Source: Authors.

2.3.2. Effect of collision's location.

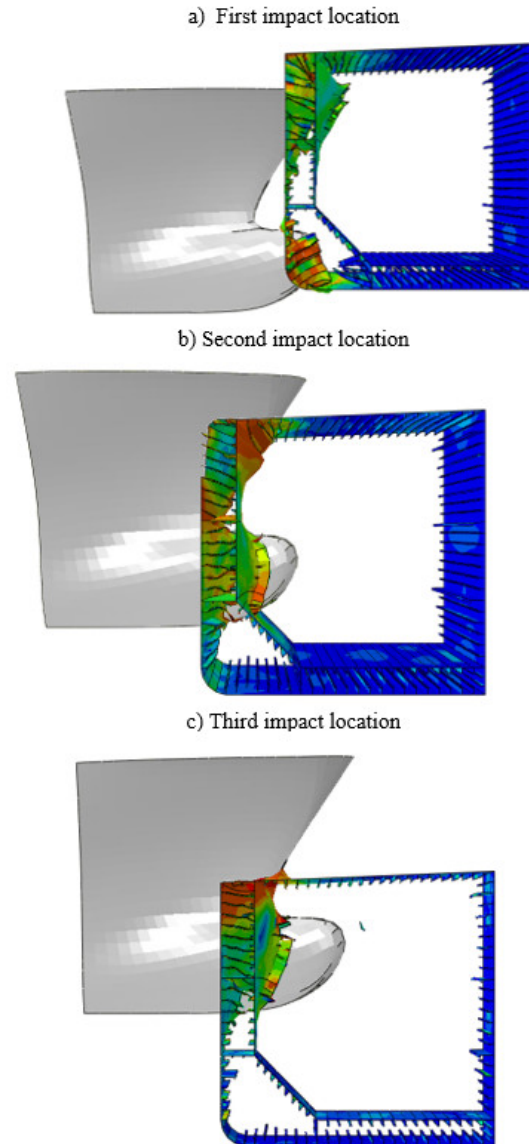
Generally, ship is build with various structural components, having different levels of strength and energy-absorbing capabilities. This section investigates the effect the collision location on the strength response of collided ship. Three impacted area were proposed, as depicted in Fig. 9. Collision angle is assumed to be 90° within the velocity of 12m/s. From graphical results, it can be seen that both outer and inner hulls were completely penetrated for different impact locations.

Fig. 10. shows a comparison between the proposed impacted regions regarding collision force versus time (Fig. 10.a), and displacement (Fig. 10.b). The deepest bow penetration was obtained at the third impact location. However, It was relatively small at the first impact location because of the strength capacity along the vertical direction of the side hull structure. It should be mentioned that the structural integrity at the first location has a superior resistance compared to the other collision impact locations. additionally, higher absorbed energy is attributed to the first location, whereas less energy is absorbed simultaneously for the second and third locations. The energy absorption capacity depends on the structural elements of the hull.

2.3.3. Effect of friction coefficient.

Friction coefficient values (0.1, 0.23, 0.3, and 0.4) were chosen according to the literature, in order to evaluate its effect on the dissipation of energy during the collision simulation. Fig. 11. shows the influence of the friction coefficient on the total absorbed energy for different ship velocities. Compared to

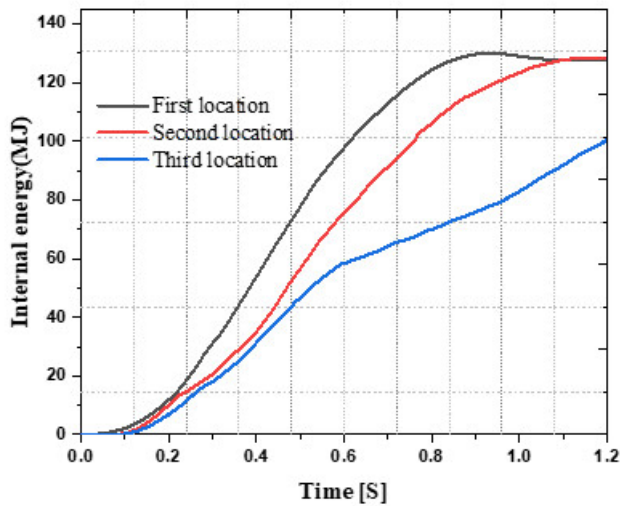
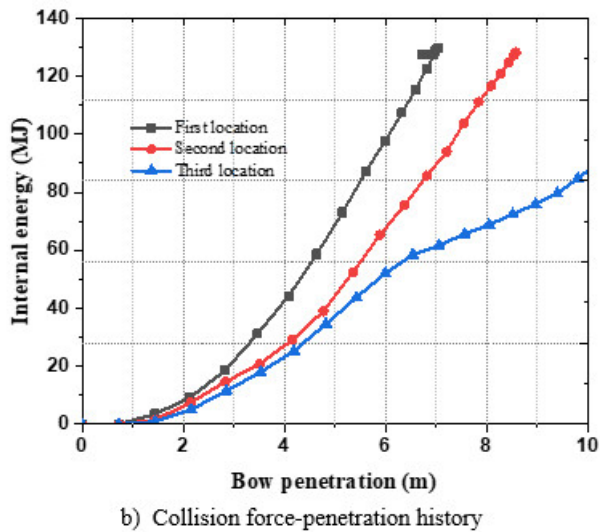
Figure 9: Structural damage for different collision locations.



Source: Authors.

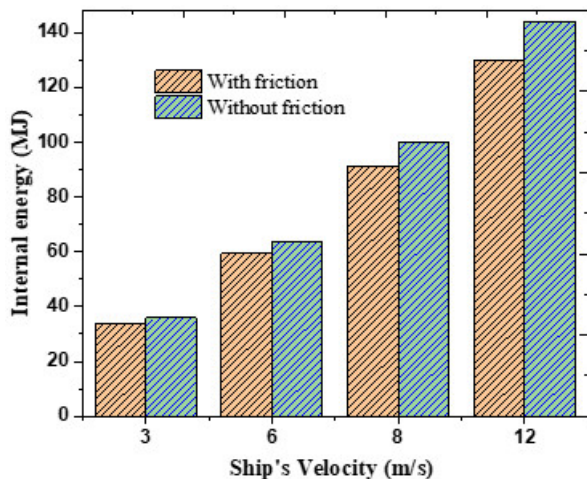
the case without friction, it is clear that the friction coefficient contributes to the total energy. Furthermore, this contribution has a significant effect when the ship's velocity increased. Fig. 12 shows that the effect of friction becomes more pronounced when bow penetration increases. However, for a small penetration (from 0 m to 3.5 m), this effect was almost negligible. With a higher friction coefficient, there was more resistance, leading to a slightly higher force at the moment of impact. The obtained results indicated that the effect of the friction coefficient is not significant compared to other collision parameters. This finding is confirmed (Wiśniewski and Kołakowski, 2003; Zhang et al., 2019)

Figure 10: Comparison of collision force data for the proposed locations.



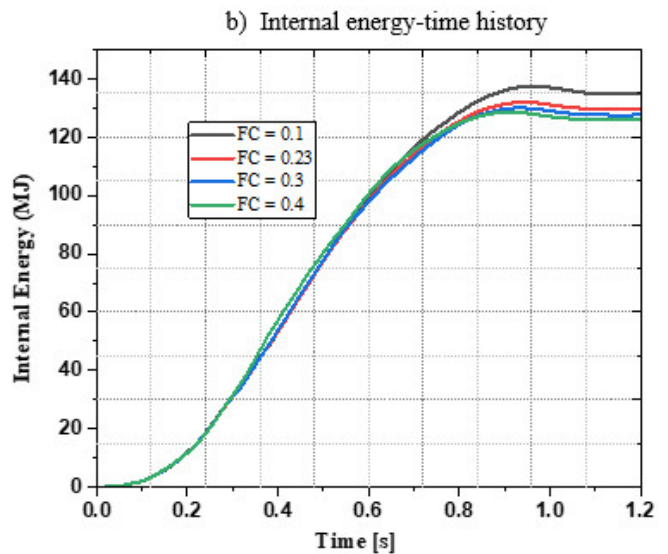
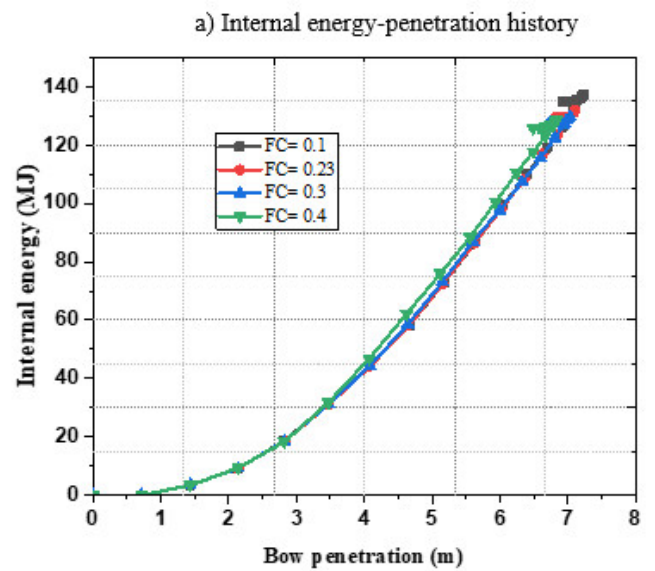
Source: Authors.

Figure 11: Contribution of friction coefficient on the absorbed internal energy for different ship's velocities.



Source: Authors.

Figure 12: Absorbed energy during a collision event for different friction coefficients.

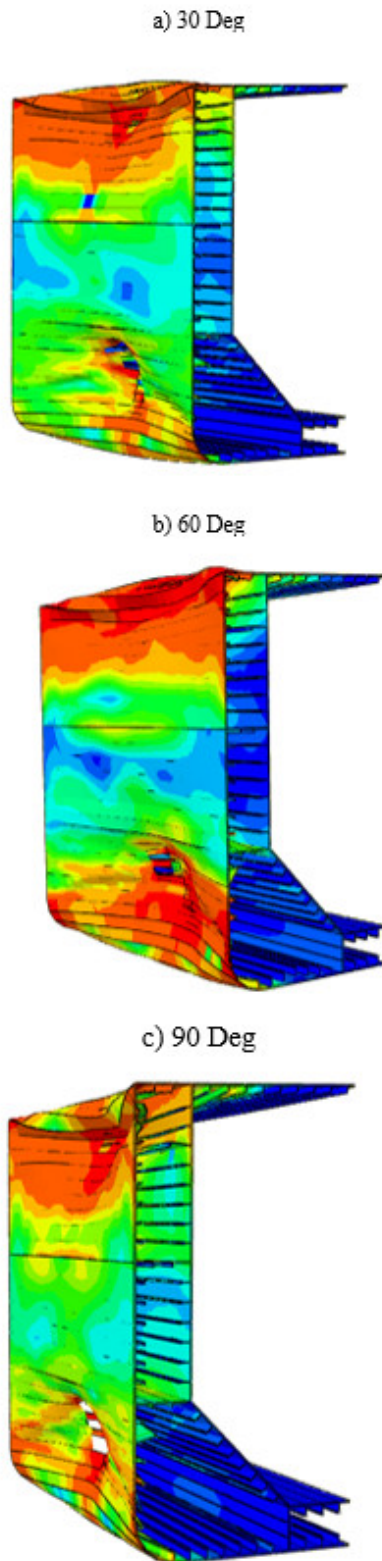


Source: Authors.

2.3.4. Effect of collision angle.

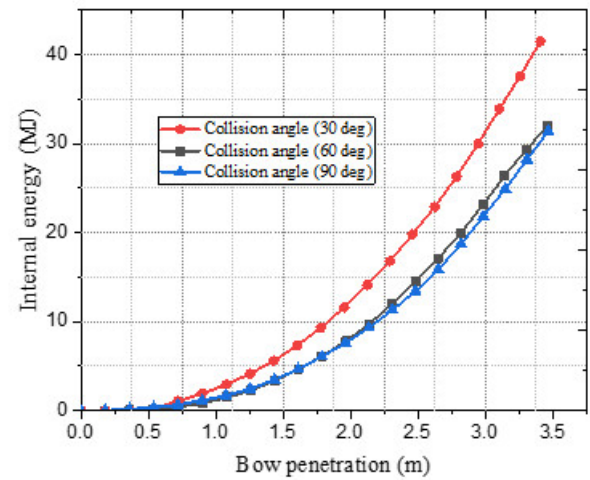
This section investigates the effect of collision angle on the structural response of a double-hull ship. Several angles were considered, varying from 30 to 150 degrees. Fig. 13 shows a cut-view of the structural damage during a ship collision at various angles. From Fig. 14, it is evident that the right angle (90 degrees) has a significant effect compared to other angles. Higher absorbed energy is attributed to the 30-degree angle. It is concluded that the angle at which the impact occurs affects the distribution of forces and energy transfer between the ships, influencing the penetration depth and the location of damage along the hulls.

Figure 13: Cut-damage view of the hull side ship for different collision angle.

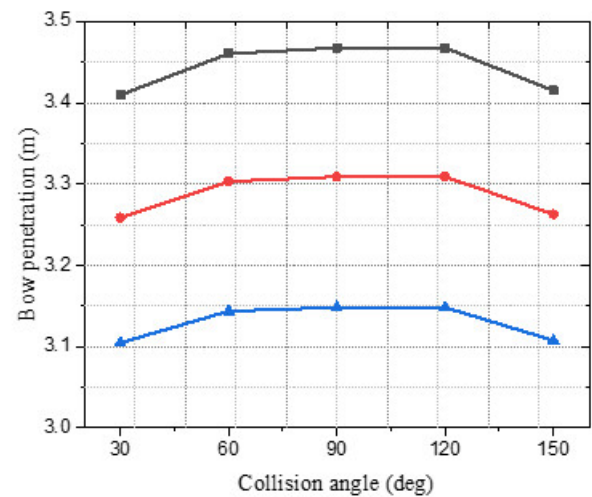


Source: Authors.

Figure 14: Effect of collision angle on the structural response of the impacted ship .



b) Bow Penetration versus collision angle



Source: Authors.

Conclusions.

The structural integrity of a ship should be enhanced to improve its resistance to accidental events. This study proposes a 3D numerical model to evaluate the crashworthiness of a double-hull tanker ship colliding with the rigid bow of a container ship. A nonlinear finite element analysis is employed to simulate the structural response of the impacted ship. To accurately capture the large deformations during a collision, plastic deformation is incorporated into the material behavior of the numerical model. Additionally, smaller elements are utilized in the impacted region of the struck ship. A parametric study is conducted to assess the influence of collision parameters on total energy dissipation, damage extent, and penetration depth. The analyzed collision variables include velocity, location, angle, and friction coefficient. Results indicate that collision velocity has the most significant impact on the hull-side

structure. The hull's structural elements contribute to overall collision resistance, with higher energy absorption observed in areas of greater structural resistance. The effect of friction coefficients is found to be minor. Collisions at a right angle (90 degrees) result in the highest penetration. The numerical findings provide insights into the structural response of double-hull ships subjected to impact loading.

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