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Collision Avoidance of Vessels by Predicting the Maximum Start Time of a Divergence Maneuver

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ABSTRACT

The issues of collision avoidance of vessels are becoming increasingly important in the contemporary world due to the increase in the number and dimensions of vessels. In situations of dangerous convergence, various types of interaction between vessels are possible. The methods of compensating for situational disturbance are selected depending on the level of its danger. For this purpose, it is necessary to build an analytical model for forming the area of inadmissible positions in such a way that its boundary corresponds to the zero probability of a vessel collision. The main characteristics of the danger are the distance and/or time of the shortest convergence. The article proposes a characteristic of the maximum time for the start of a divergence maneuver, which determines the possibility of its safe implementation. A method for predicting this maximum start time of a divergence maneuver is considered taking into account the ratio of the speeds of the maneuvering vessel and the target. Analytical model for the implementation of the proposed method is given.
The proposed method provides the operator of the vessel traffic management system (VTMS) with the opportunity to make timely decisions to collision avoidance of vessels. In addition, automatization of VTM can provide significant assistance for the development of future unmanned shipping, ensuring safety navigation.

1. Introduction.

A feature of modern navigation is the steady increase in the volume of cargo transportation by vessels of various sizes and purposes [1].

In such conditions, maneuvering becomes increasingly complex, and bridge personnel have to predict the movement of neighboring ships and make decisions to collisions avoidance in increasingly shorter periods of time. Therefore, the issues of collision avoidance of vessels are considered in numerous scientific publications from various theoretical positions and taking into account the specifics of practical experience of navigation in various basins of the world [2; 3].

A method for forming flexible strategies for avoiding a vessel with several dangerous targets using locally independent control methods and fuzzy logic methods, which takes into account the requirements of COLREG-72, navigational hazards and vessel dynamics, are proposed in [4; 5].

The paper [6] examines the types of interaction between

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vessels of different types in various situations of dangerous convergence and methods of compensating for situational disturbances depending on the level of its danger.

The principles of locally independent and remote control of the process of divergence of dangerously converging vessels are considered in the work [7]. It also provides an analysis of the methods of their implementation and considers promising current methods of increasing the safety and collision avoidance of vessels.

The collision avoidance between vessels in a situation of their excessive proximity is studied in the work [8], which also proposes a strategy for emergency divergence.

In works [9; 10], a method is proposed for taking into account navigational risks in the maneuvering area and the inertia of the vessel when calculating the parameters of its divergence strategy, and in work [11], a method is presented for selecting the optimal maneuver for diverging a pair of vessels by changing course.

The publication [12] indicates that the vessel traffic management system (VTMS), as a rule, does not have the technical capabilities to control the movement of vessels in areas of their accumulation. To avoid collisions, a new fuzzi method is proposed in the article. By using an analytical model of the marine geographic information system (GIS), an accurate prediction of the collision time and position can be obtained. The proposed method gives the VTMS operator the opportunity to make decisions to prevent collisions of vessels.

In [13], the process of divergence of courts is formalized in terms of a differential game. A comparison of five algorithms for determining the safe trajectory of the vessel in a collision risk situation is carried out; the results are illustrated by examples of computer modeling of algorithms for determining safe and optimal trajectories of own vessel in real navigation situations at sea.

In the works [14; 15] it is shown that research on ship control automation can be presented by a classical approach based on mathematical models and algorithms. Computer technologies using artificial intelligence (AI), neural networks (NN) or their combination in hybrid systems can also be used [16].

The aim of the article is to develop a method for predicting the maximum start time of a divergence maneuver.

2. Mathematical model for areas of unacceptable positions.

2.1. Collision hazard characteristics.

Each of the vessels is associated with a two-dimensional region S_{nd}, called the region of inadmissible positions (domain), into which it is undesirable for any foreign objects to enter. The region of inadmissible positions S_{nd} is formed in such a way that its boundary corresponds to a zero probability of vessel collision.

The risk of collision arises when a vessel is predicted to enter the area of inadmissible positions S_{nd} of the partner. It indicates the upcoming inadmissible situation in advance, based on the forecast of the change in the relative position of the pair of vessels. The risk of collision is conditional, since its truth is affected by the possible actions of the vessels before reaching the inadmissible situation. Obviously, the risk of collision arises when the predicted value of the closest approach distance D_{min} is less than the value of the maximum permissible approach distance D_d, the value of which depends on the shape of the area of inadmissible positions and the angle of the converging vessels.

The second characteristic of the collision risk is the time of closest convergence, i.e. the moment of time of achieving the closest convergence. However, as a more informative characteristic concerning the divergence process, we can propose the maximum start time of the divergence maneuver t_{di}, the meaning of which has the following interpretation. If, in the presence of a collision hazard, the distance between the vessels exceeds the value D_d, and the operating vessel can ensure the maximum value of the closest approach distance $\max D_{min}$ by its maneuver, for which $\max D_{min} > D_d$, then it is in an acceptable position. Over time, with unchanged parameters of the vessels' movement, the distance between them decreases and the moment of time t_{di} occurs when the equality $\max D_{min} = D_d$ is achieved.

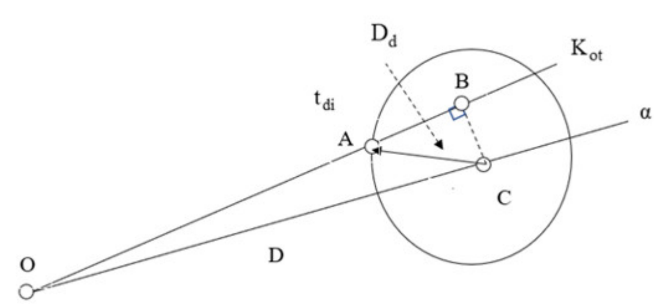
With further approach of vessels with program parameters of movement $\max D_{min} < D_d$, the operating vessel falls into a subset of unacceptable positions, and no course change maneuver will allow it to pass the target at a distance of D_d.

2.2. Analytical expressions for calculating the start time moment of maneuvering.

2.2.1. The speed of the operating vessel is greater than the speed of the target.

Let us define an analytical expression for calculating the moment of time t_{di} when the operating vessel avoids a collision by changing course, taking into account the ratio of the speeds of the operating vessel and the target. First, let us consider the case when the speed of the operating vessel is greater than the speed of the target, i.e. $\rho \geq 1$ (Fig. 1). Obviously, in this case the moment of time t_{di} is determined when the operating vessel reaches the boundary of the region of unacceptable positions of the target, and the equality $D(t_{di}) = D_d$ is valid.

Figure 1: To determine t_{di} at $\rho \geq 1$.



Source: Authors.

where:
O = Operating vessel.
C = Target.

t_{di} = Estimated start time of the divergence maneuver.
$D(t_{di}) = D_d$ = Distance the target goes to the encounter.
K_{ot} = Initial direction of vessel movement.
α and D = Initial values of bearing and distance to the target.

In work [14], the dependence of the current distance *D(t)* on time *t* was obtained with constant parameters of relative motion K_{ot} and V_{ot}

$$D(t) = \sqrt{V_{ot}^2 t^2 + D^2 - 2DV_{ot}t\cos(K_{ot} - \alpha)}$$

where K_{ot} – initial relative course, degrees;
V_{ot} – initial relative velocity, knots.
We can write the equation:

$$D(t_{di}) = \sqrt{V_{ot}^2 t_{di}^2 + D^2 - 2DV_{ot}t_{di}\cos(K_{ot} - \alpha)} = D_d$$

from which it follows:

$$V_{ot}^2 t_{di}^2 + D^2 - 2DV_{ot}t_{di}\cos(K_{ot} - \alpha) = D_d^2$$

We rewrite the last equation in the following form:

$$V_{ot}^2 t_{di}^2 - 2DV_{ot}t_{di}\cos(K_{ot} - \alpha) + D^2 - D_d^2 = 0$$

or

$$t_{di}^2 - 2kt_{di} + q = 0, \quad (1)$$

where:

$$k = \frac{D\cos(K_{ot} - \alpha)}{V_{ot}}$$

$$q = \frac{D^2 - D_d^2}{V_{ot}^2}$$

Solving equation (1) for the variable t_{di} we obtain:

$$t_{di,1,2} = k \pm \sqrt{k^2 - q}$$

Substituting the values of k and q, we find the roots t_{di1} and t_{di2}:

$$\begin{aligned} t_{di1} &= \frac{D\cos(K_{ot} - \alpha)}{V_{ot}} + \sqrt{\frac{D^2\cos^2(K_{ot} - \alpha)}{V_{ot}^2} - \frac{D^2 - D_d^2}{V_{ot}^2}} \\ &= \frac{1}{V_{ot}}\left\{D\cos(K_{ot} - \alpha) + \sqrt{D_d^2 - D^2[1 - \cos^2(K_{ot} - \alpha)]}\right\} \\ or \quad &= \frac{1}{V_{ot}}\left\{D\cos(K_{ot} - \alpha) + \sqrt{D_d^2 - D^2\sin^2(K_{ot} - \alpha)}\right\} \\ t_{di2} &= \frac{1}{V_{ot}}\left\{D\cos(K_{ot} - \alpha) - \sqrt{D_d^2 - D^2\sin^2(K_{ot} - \alpha)}\right\} \end{aligned}$$

Please note that the moment of time t_{di} of reaching the boundary of the area of unacceptable positions by the operating vessel corresponds to the root t_{di2} of equation (1) – point A in Fig. 1. Therefore:

$$t_{di} = \frac{1}{V_{ot}}\left\{D\cos(K_{ot} - \alpha) - \sqrt{D_d^2 - D^2\sin^2(K_{ot} - \alpha)}\right\} \quad (2)$$

The correctness of the obtained analytical expression for the calculation is confirmed by the analysis of Fig. 1, from which it follows that:

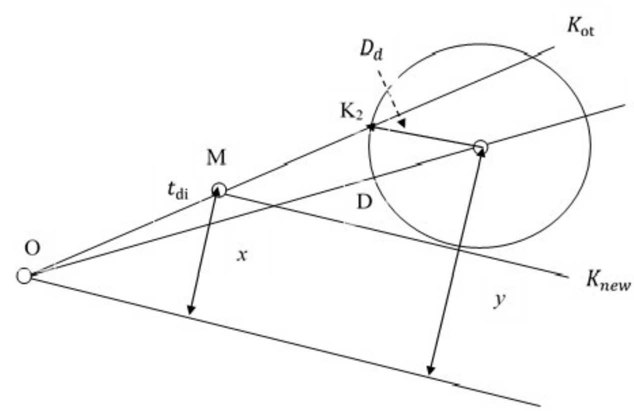
$$t_{di} = \frac{OA}{V_{ot}}$$

In turn, OA = OB – – AB, with OB=D × $\cos(K_{ot} - \alpha)$ and AB= $\sqrt{D_d^2 - D^2\sin^2(K_{ot} - \alpha)}$. Therefore, formula (2) is correct.

2.2.2. *The speed of the operating vessel is less than the speed of the target.*

If the speed of the operating vessel is less than the speed of the target, i.e. $\rho < 1$, then to derive the expression for calculating the value of t_{di} we will use Fig. 2. To determine the moment of time t_{di}, i.e. the turn to the extreme evasion course, we will use Fig. 2.

Figure 2: To determine t_{di} at $\rho < 1$.



Source: Authors.

From Fig. 2 it follows:

$$t_{di} = \frac{OM}{V_{ot}}$$

The desired value OM is determined from the expression:

$$OM = \frac{x}{\sin(K_{ot} - K_{new})}.$$

Course K_{new} is the relative extreme course at which the maximum value of the closest convergence distance maxD_{min} is achieved, and

$$K_{new} = p + K_2 \pm \arcsin\rho,$$

where $\rho = V_1/V_2$ and $V_2 > V_1$; K_2 – target course.
In turn,
$x = y - D_d$ and $y = \Delta_y D \times \sin(K_{new} - a)$,
where $\Delta_y = \mathrm{sign}[\sin(K_{new} - K_{ot})] = \pm 1$.
So,

$$t_{di} = \frac{\Delta_y D\sin(K_{new} - \alpha) - D_d}{\Delta_y V_{otn}\sin(K_{ot} - K_{new})} \quad (3)$$

Thus, for the case $\rho < 1$, the calculation of the moment of time t_{di} is performed using expression (3).

Consequently, in the case of choosing a divergence maneuver by the changing the operating vessel course, the moment of time t_{di} is calculated using the expression (4):

$$t_{di} = \begin{cases} \dfrac{D\cos(K_{ot}-\alpha) - \sqrt{D_d^2 - D^2\sin^2(K_{ot}-\alpha)}}{V_{ot}}, & \rho \geq 1; \\ \dfrac{\Delta_y D\sin(K_{new}-\alpha) - D_d}{\Delta_y V_{ot}\sin(K_{ot}-K_{new})}, & \rho < 1. \end{cases} \quad (4)$$

3. Results.

The task of choosing the optimal maneuver is very complex, since the process of controlling the ship's motion is rich in nonlinear and non-stationary characteristics, and the task is to be of a gaming nature. The research is of a theoretical nature, considering the possibility of describing the vessels divergence process and its forward modeling. The initial data for $\rho = 1$ (ver. 1) are presented in Table 1. Variable characteristics for modeling the calculation process are marked in green.

Table 1: The initial data (ver. 1).

Type	Initial distance, miles	Speed, knots	Time to collision, hours	Time to collision, minutes	Distance to collision place, miles	Direction, degrees
Vessel	0	10	0,77	46,15	7,69	40,00
Target	10	3	0,77	46,15	2,31	35,00

Source: Authors.

The application of the obtained formulas (2) gives, with the specified initial data, the resulting time for the operating vessel to reach the boundary of the area of unacceptable positions. For the specified data, we have the entry time and exit time within the target:

$t_1 = 0{,}78$ h;

$t_2 = 1{,}21$ h.

The calculation of the change course angles from the initial one is given in Table 2. It is obvious that when approaching the moment of meeting, the angles become larger and less convenient for performing the maneuver.

On Table 2:

Green = Optimal angle.

Yellow = Warning angle.

Orange = Collision risk angle.

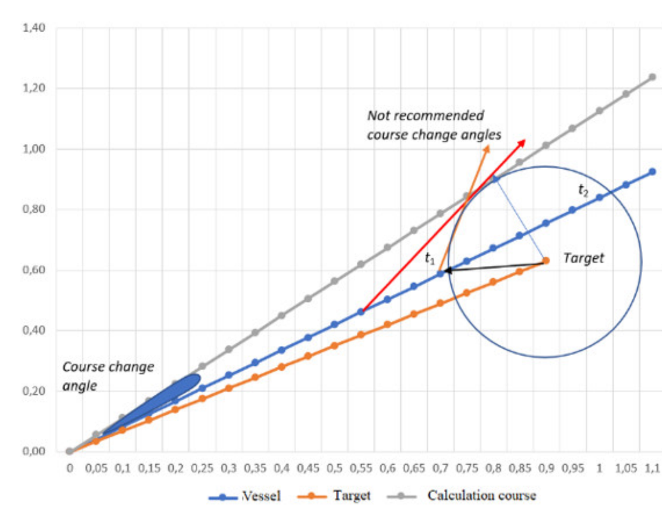
Taking into account the recommended restrictions on the change of the vessel course in 0–20^o, in order to collision avoidance, we see the division into three zones of action (Fig. 3).

Table 2: Angle of course change at different moments of time.

Time, hours	Distance, miles	Course change angle, degrees
0	10	8,34
0,05	9,5	9,06
0,1	9	9,86
0,15	8,5	10,75
0,2	8	11,77
0,25	7,5	12,92
0,3	7	14,25
0,35	6,5	15,80
0,4	6	17,62
0,45	5,5	19,81
0,5	5	22,49
0,55	4,5	25,85
0,6	4	30,23
0,65	3,5	36,25
0,7	3	45,28
0,75	2,5	62,38

Source: Authors.

Figure 3: Modeling of a change in the operating vessel course (ver. 1).



Source: Authors.

For the initial data for ρ = 1 (ver. 2), presented in Table 3, the results of modeling a change in the operating vessel course are shown in Table 4 and Fig. 4.

Table 3: The initial data (ver. 2).

Type	Initial distance, miles	Speed, knots	Time to collision, hours	Time to collision, minutes	Distance to collision place, miles	Direction, degrees
Vessel	0	16	0,79	47,37	12,63	33,00
Target	15	3	0,79	47,37	2,37	40,00

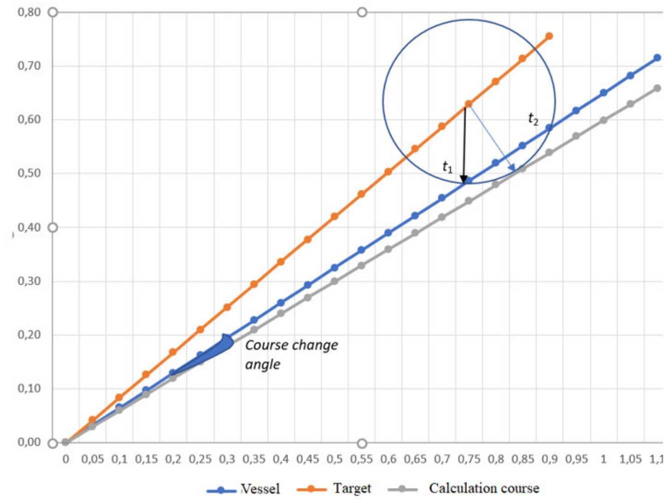
Source: Authors.

Table 4: Angle of course change at different moments of time.

Time, hours	*Distance, miles*	*Course change angle, degrees*
0	15,0	2,08
0,05	14,2	2,60
0,1	13,4	3,18
0,15	12,6	3,83
0,2	11,8	4,58
0,25	11,0	5,43
0,3	10,3	6,43
0,35	9,5	7,59
0,4	8,7	8,99
0,45	7,9	10,68
0,5	7,1	12,78
0,55	6,4	15,46
0,6	5,6	19,01
0,65	4,9	23,99
0,7	4,1	31,56
0,75	3,4	45,14

Source: Authors.

Figure 4: Modeling of a change in the operating vessel course (ver. 2).



Source: Authors.

For the specified data, we have the time of crossing the safety limits: t_1= 0,84 h; t_2= 1,02 h.

The initial data for ρ < 1 are presented in Table 5.

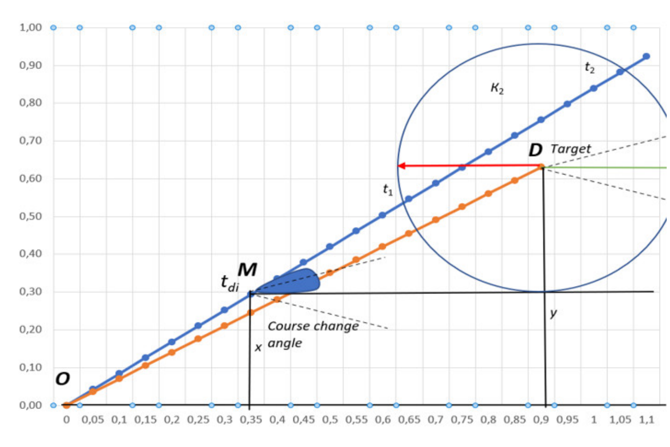
Table 5: The initial data for ρ? < 1.

Type	Initial distance, miles	Speed, knots	Time to collision, hours	Time to collision, minutes	Distance to collision place, miles	Direction, degrees
Vessel	0	5	1,33	80,00	6,67	40,00
Target	20	10	1,33	80,00	13,33	35,00

Source: Authors.

The results of modeling a change in the operating vessel course for ρ < 1 are shown in Fig. 5.

Figure 5: Modeling of a change in the operating vessel course for ρ < 1.



Source: Authors.

Let's calculate the new vessel course K_{new}, assuming that the target course is K_2=160^o:

$$K_{new} = 180^\circ + K_2 \pm \arcsin\rho = \begin{cases} 10^\circ \\ 310^\circ \end{cases}$$

where $\rho = V_1/V_2$ and $V_2 > V_1$; K_2 – target course.

We substitute in (4) and get the time t_{di} before the start of the U-turn maneuver:

$$t_{\text{di}} = \frac{(-1)\ \times 20 \times \sin(310^o - 35^o)\ \ - - 13,33}{(-1)\ \times 5 \times \sin(40^o - 310^o)} = 1,32\ (hours)$$

At the same time, the course change is quite significant (by 90°), and it can be performed only by reducing the vessel speed. However, this value is calculated for an extreme convergence, which is not optimal, since the maneuver at the very beginning of target identification is more favorable, with a course deviation from 40° to 10°.

4. Discussion.

The generalization of changing the ship's direction is given in Table 6.

Table 6: Changing the vessel course direction.

Time, hours	*Distance, miles*	Changing the course direction	*Time, hours*	*Distance, miles*	Changing the course direction
0	20	30,00	0,65	16,75	61,20
0,05	19,75	32,22	0,7	16,50	63,60
0,1	19,5	34,44	0,75	16,25	66,00
0,15	19,25	36,67	0,8	16,00	68,40
0,2	19	38,89	0,85	15,75	70,80
0,25	18,75	41,11	0,9	15,50	73,20
0,3	18,5	43,33	0,95	15,25	75,60
0,35	18,25	45,56	1	15,00	78,00
0,4	18	47,78	1,05	14,75	80,40
0,45	17,75	50,00	1,1	14,50	82,80
0,5	17,5	52,22	1,15	14,25	85,20
0,55	17,25	54,44	1,2	14,00	90,00
0,6	17	56,67	1,25	13,75	85,56
0,65	16,75	58,89	1,3	13,5	90,00

Source: Authors.

Values colored blue in Table 6 indicate conditions that are not suitable for safe navigation and are emergency, and the vessel may be damaged via heeling or otherwise.

If the divergence maneuver is performed by the operating vessel braking, then the time moment t_{di} is calculated for the situation when the vessel stops as a result of braking, and the closest approach distance is equal to the maximum permissible distance. Obviously, at the moment the operating vessel stops, the relative course is equal to the reverse course of the target, which is stationary in relative motion, i.e. $K_{\text{ot}} = K_2 + 180$, and the relative speed is equal to the speed of the target. Taking into account the above, expression (3) can be used to calculate t_{di}, taking $K_{\text{new}} = K_{\text{ot}}$:

$$t_{\text{di}} = \frac{\Delta_y \times D \times \sin(K_2 + 180 - \alpha) - D_d}{\Delta_y \times V_{ot} \times \sin(K_{ot} - K_2 - 180)}$$

The expressions obtained for calculating the time t_{di} do not take into account the vessel dynamics. Therefore, taking into account the maneuvering vessel dynamics, the value of the moment of time t_{di} is determined by the expression:

$$t^*_{\text{di}} = t_{\text{di}} - \Delta t\ ,$$

where Δt is the time correction via the vessel dynamics.

In some cases, instead of changing the course, it is reasonable to wait until the target moves to a safe distance without changing the operating vessel direction. In this case, it is necessary to determine the time of the extreme stop of the vessel. Let's calculate for the above conditions:

$$t_{\text{di}} = \frac{\Delta_{\text{y}} \times D \times \sin(K_2 + 180 - a)\ -\ D_d}{\Delta_y \times V_{ot} \times \sin(K_{ot} - K_2 - 180)} = 0,70\ \ (hours)\,.$$

An extreme stop of the vessel can also be performed at an arbitrary moment of time up to 0,7 h for a safer situation. After passing the target to a safe area, the vessel movement is resumed in the previous course.

Conclusions.

- It is proposed using the maximum start time of the divergence maneuver to ensure its safe navigation.
- A method for predicting this time is proposed, taking into account the ratio of the speeds of the operating vessel and the target.
- Analytical expressions are given for the implementation of the proposed method.

References.

1. Hahn, T., Rethfeldt, C. & Jeinsch, T. (2022). Reference tracking control for maneuvering vessels considering input delay. *IFAC-PapersOnLine*. 55, 445–450. https://doi-.org/10.1016/j.ifacol.2022.10. 468.
2. Zhaoxu, T., Cheng, Yo., Yao, S. & Li, Z. (2024). An adaptive INS/CNS/SMN integrated navigation algorithm in sea area. *Remote Sensing*, 16(4):612. https://doi.org/10-.3390/rs16040612.
3. Gladkykh, I., Golikov, A., Vorokhobin, I. & Kulakov, M. Development prospects of the Ukrainian section of the shipping route E-40. *Proc. of the 24th Internat. Sc. Conf. on Transport Means* [online], Kaunas, Lithuania, Sept. 30 – Oct. 02, 2020, 860–864. Available from: https://transportmeans.ktu.edu/wp-content/ uploads/ sites-/307/2018/02/Transport-means-A4-II-dalis.pdf [Accessed 8 September 2024].

4. Brcko, T. & Luin, B. (2023). A decision support system using fuzzy logic for collision avoidance in multi-vessel situations at sea. *Journal of Marine Science and Engineering*, 11(1819). https://doi.org/10.3390/jmse11091819.
5. Xie, W., Gang, L., Zhang M., Liu T. & Lan Z. (2024). Optimizing multi-vessel collision avoidance decision making for autonomous surface vessels: A COLREGs - compliant deep reinforcement learning approach. *Journal of Marine Science and Engineering*, 12, 372. https://doi.org/10.3390/jmse12030372.
6. You, J., Jia, Z., Dong, C., He, L., Cao, Yi., & Wu, Q. (2023). Computation offloading for uncertain marine tasks by cooperation of UAVs and vessels. *Preprint,* arXiv.2302.06055v1. 6 p.
7. Sun, T., Liu, C., Xu, S., Hu, Q. & Li, C. C (2024). COLREGS −complied automatic collision avoidance for the encounter situations of multiple vessels. *Journal of Marine Science and Engineering,* 10, 1688, 1–22. https://doi.org/ 10.3390/jmse10111688.
8. Pipchenko, O., Konon, N. & Bogachenko, Y. (2023). Mathematical modelling of "asd tug – marine vessel" interaction considering tug's maneuverability and stability limitations. *Journal of Maritime Research*, 20 (2), 117–124. Available from: https://www.jmr.unican.es/index.php/jmr/article/view/722 [Accessed 9 September 2024].
9. Imazu, H. (2017). Evaluation method of collision risk by using true motion. *TransNav. International Journal on Marine Navigation and Safety of Sea Transportation*, 11, 1, 65–70. https://doi.org/ 10.12716/1001.11.01.06.
10. Burmaka, I., Vorokhobin, I., Vorokhobin, M. & Zhuravska, I. (2022). Forming the area of unacceptable values of the parameters of vessels' movement for the vessels' divergence at remote control process. *Acta Innovations*, 44, 5–17. https://doi.org/10.32933/ActaInnovations.44.1.
11. Mallampalli1, P., Janardhanan, S. & Kesavadev.V. K. (2023). Numerical study on the maneuvering of a container ship in shallow water waves. *Journal of Maritime Research*, 20 (2), 125–134. Available from: https://www.jmr.unican.es/ index.php/jmr/article/view/744/746 [Accessed 9 September 2024].
12. Uchacz, W. & Pietrzykowski, Z. (2003). Vessel traffic optimization using a linear model with fuzzy coefficients. *In: Sołdek, J., Drobiazgiewicz, L. (eds) Artificial Intelligence and Security in Computing Systems. The Springer International Series in Engineering and Computer Science*, 752, 23–32. Springer, Boston, MA. https://doi.org/10.1007/978-1-4419-9226-0_3.
13. Lisowski, J. (2014). Game strategies of ship in the collision situations. *TransNav the International Journal on Marine Navigation and Safety of Sea Transportation*, 8 (1), 69–77. https://doi.org/ 10.12716/1001.08.01.08.
14. Vorokhobin, I., Zhuravska, I., Burmaka, I. & Kulakovska, I. (2021). Analysis of the error distribution density convergence with its orthogonal decomposition in navigation measurements. *Journal of Physics: Conference Series*, 2090 (1), 012126. https://doi.org/10.1088/1742-6596/2090/1/012126.
15. Vorokhobin, I., Haichenia, O., Sikirin, V. & Severin, V. (2020). Determination of the law of probability distribution of navigation measurements. *Proc. of the 24th Internat. Sc. Conf. on Transport Means* [online], Kaunas, Lithuania, Sept. 30 – Oct. 02, 2020, 707–710. Available from: https://transportmeans.ktu.edu/wp-content/uploads/sites/307/2018/02/Transport-means-A4-II-dalis.pdf [Accessed 8 September 2024].
16. Statheros, T., Howells, G. & McDonald-Maier, K. (2007). Autonomous ship collision avoidance navigation concepts, technologies and techniques. *Journal of Navigation,* 61 (01), 129–142. https://doi.org/10.1017/S037346330700447X.