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MATHEMATICAL MODELS OF SHIPS FOR MANOEUVRING SIMULATION AND CONTROL

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ABSTRACT

This paper reviews models with four degrees of freedom, including the roll motion, in order to describe the movement of a ship. The effects of yaw, sway and roll are also taken into account. These models are used in applications such as the simulation and steering control of a ship, or the study of sea vessels which act cooperatively.

Key Words: Mathematical model. Ship movement. Manoeuvring. Simulation.

INTRODUCTION

Computer simulation, which started in the fifties, is now the most widely used method for evaluating the manoeuvrability of conventional surface ships, submarines and other craft (Barr, 1993). Simulation is also a widely accepted tool used in ship design and research, selection and design of ship equipment, design and research of waterways and harbours and training of ship officers.

The simulation methods use hydrodynamic coefficients based on data obtained using captive model tests performed in a towing tank and/or a rotating arm. In marine science, time domain simulations have been used mainly to predict the controllability and manoeuvrability of all types of sea craft and systems.

For any control design, knowledge of the dynamic characteristics of the device or physical system to be controlled is essential. In the manoeuvring and control of ships, experience suggests that it is difficult to predict the characteristics of a ship from model tests, due to the lack of any exact knowledge of the steering and roll interactions (Blanke and Jensen, 1997). Hence, a great deal of research has been carried out in order to analyse this interaction (coupling). Knowledge of the dynamics related to yaw, sway and roll is useful both for improving manoeuvring models and, for example, developing roll damping control applications in which the dynamic couplings between the yaw, sway and roll are of great importance.

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Validation of an autopilot system is performed first by simulation, then by model and full-scale trials.

While models of three degrees of freedom for describing ship motion are well-known (Abkowitz, 1964 and Chislett and Støm-Tejsen, 1965), there have been very few studies describing the coupling between yaw, sway and roll. Results published by Son and Nomoto (1981) present the model of a container ship obtained by combining planar motion mechanism (PMM) data for lateral movements using different values of static heel for the model under test, with independent roll motion tests. Kalstrom and Otterson (1983) obtained a model by combining a lateral PMM model with theoretical estimates of roll coefficients, using free sailing model tests to calibrate the roll parameters. Blanke and Jensen (1997) obtain a full non-linear model of a container ship using the unique four degrees of freedom roll planar motion mechanism (RPMM) facility at the Danish Maritime Institute, and the model has also been validated via extensive full scale trials.

This paper describes models of four degrees of freedom used for simulation and control applications, presenting the hydrodynamic models of two container ships (Son and Nomoto, 1981) and (Blanke and Jensen, 1997).

SHIP MATHEMATICAL MODEL

Degrees of Freedom and Notations

The movement of a ship, considered as a rigid solid, has six degrees of freedom (DOF) which means that six independent coordinates are required to determine its position and orientation. Table 1 shows the description of each DOF and the corresponding nomenclature used to describe the ship's forces and motions. This is the standard notation recommended in SNAME, (1950) for use in ship manoeuvring and control applications.

Table 1: Notation and DOF description

DOF	Translation	Forces	Linear velocity	Position
1	surge	X	u	x
2	sway	Y	v	y
3	heave	Z	w	z
	Rotations	**Moments**	**Angular velocity**	**Angles**
4	roll	K	p	ϕ
5	pitch	M	q	θ
6	yaw	N	r	ψ

The first three coordinates and their derivatives are used to describe the position and translation movements on the axes x_B, y_B and z_B , while the other three coordinates and their derivatives are used to describe the orientation and rotation movements. For sea vessels, the six different motion components are defined as surge, sway and heave for translation motions in the three directions and roll, pitch and yaw for rotation motions around the three axes.

Using the above notation, the motion of a ship with six DOFs can be described by means of the following vectors:

• The speed vector which is normally defined in relation to the ship's coordinates system:

$$\mathbf{v} = \left[\mathbf{v}_1^T, \mathbf{v}_2^T\right]^T, \quad \mathbf{v}_1 = \left[u, v, w\right]^T, \quad \mathbf{v}_2 = \left[p, q, r\right]^T \tag{1}$$

• The external forces and motion vector which is also defined in relation to the ship's coordinates system:

$$\boldsymbol{\tau} = \left[\boldsymbol{\tau}_1^T, \boldsymbol{\tau}_2^T\right]^T, \quad \boldsymbol{\tau}_1 = \left[X, Y, Z\right]^T, \quad \boldsymbol{\tau}_2 = \left[K, M, N\right]^T \tag{2}$$

• The position and orientation vector defined with respect to the inertial reference system:

$$\boldsymbol{\eta} = \left[\boldsymbol{\eta}_1^T, \boldsymbol{\eta}_2^T\right]^T, \quad \boldsymbol{\eta}_1 = \left[x, y, z\right]^T, \quad \boldsymbol{\eta}_2 = \left[\phi, \theta, \psi\right]^T \tag{3}$$

Coordinate frames

In order to analyse the ship's motion at sea in six DOF, two coordinate frames are used, as shown in Figure 1.

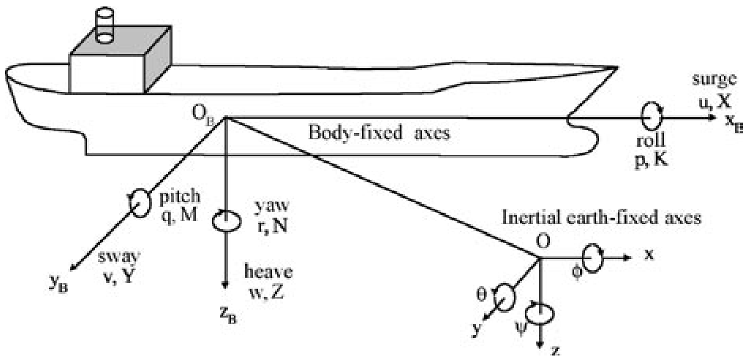


Figure 1 Coordinate Systems with definition of angles, velocities, forces and moments

The moving coordinate frame x_B, y_B, z_B is fixed to the ship and is called the body-fixed reference frame. The origin O_B of the ship's coordinate system can be selected to coincide with the Centre of Gravity (CG) if the CG is situated on the main plane of symmetry. Generally, however, this is not a good choice, since the CG is not located at any fixed point as the load conditions of the ship change constantly. The most widely used option, allowing a reduction in the complexity of the equation (Fossen, 1994), consists in selecting an orthogonal coordinate system parallel to the main axes of inertia in order to eliminate the products of inertia in the motion equations. These requirements are satisfied in practically all sea vessels, the origin being located at the intersection of the two planes of symmetry.

The motion of the body-fixed frame is described in relation to an inertial reference axis xyz, and it is normally assumed that the acceleration of one point of the surface of the Earth will have little effect on the slow motion of sea vessels. As a result, it can be considered that a reference system located on Earth O_{xyz} is an inertial system. Thus, the position and orientation of the ship is described in relation to the inertial reference system and its linear and angular velocities in the ship's mobile coordinate systems.

Ship Motion Equations

Representing the motion equations in the body-fixed reference frame, with the origin of the coordinates located at the intersection of the planes of symmetry, the motion equations of a ship, starting from Newton's equations, can be expressed (Norrbin, 1970), (Blanke, 1981) or (Fossen, 1994) as follows:

$$\begin{aligned} m\left(\dot{u} - rv + qw - (q^2 + r^2)x_G + (pq - \dot{r})y_G + (rp + \dot{q})z_G\right) &= X \\ m\left(\dot{v} - pw + ru - (r^2 + p^2)y_G + (qr - \dot{p})z_G + (pq + \dot{r})x_G\right) &= Y \\ m\left(\dot{w} - qu + pv - (p^2 + q^2)z_G + (rp - \dot{q})x_G + (qr + \dot{p})y_G\right) &= Z \end{aligned} \quad \textbf{(4)}$$

$$\begin{aligned} I_x\dot{p} + (I_z - I_y)rq + m\left(y_G(\dot{w} + pv - qu) - z_G(\dot{v} + ru - pw)\right) &= K \\ I_y\dot{q} + (I_x - I_z)rp + m\left(z_G(\dot{u} + qw - rv) - x_G(\dot{w} + pv - qu)\right) &= M \\ I_z\dot{r} + (I_y - I_x)pq + m\left(x_G(\dot{v} + ru - pw) - y_G(\dot{u} + qw - rv)\right) &= N \end{aligned} \quad \textbf{(5)}$$

where (x_G, y_G, z_G) is the position of the ship's CG; m is the mass of the ship; $u, v, w, \dot{u}, \dot{v}, \dot{w}$ represent the linear velocities and accelerations in the x_B, y_B and z_B directions; $r, q, p, \dot{r}, \dot{q}, \dot{p}$ represent the angular velocities and accelerations related to the axes x_B, y_B and z_B. I_x, I_y and I_z are the moments of inertia of the ship with respect to the same axes of the body-fixed frame. The forces and moments X, Y, Z, K, M and N represent the results of all external actions on the ship's body.

These equations can be expressed more concisely in vectorial form by the equation:

$$\boldsymbol{M}_{RB}\,\dot{\boldsymbol{v}} + \boldsymbol{C}_{RB}(\boldsymbol{v})\,\boldsymbol{v} = \boldsymbol{\tau}_{RB} \quad \textbf{(6)}$$

where $\boldsymbol{M}_{RB}$ is the inertial matrix, and $\boldsymbol{C}_{RB}(\boldsymbol{v})$ is the matrix of Coriolis and centripetal terms, both caused by the dynamics of the rigid solid. τ_{RB} is a generalised vector of the external motions and forces. These forces and motions can be broken down into several components according to their origin (Lewis, 1989), (Faltisen, 1990):

$$\boldsymbol{\tau}_{RB} = \boldsymbol{\tau}_H + \boldsymbol{\tau}_{CS} + \boldsymbol{\tau}_P + \boldsymbol{\tau}_E \quad \textbf{(7)}$$

τ_H: hydrodynamic forces and motions produced by the movements of the hull in the water, normally separated according to their origin into several groups using the equation $\tau_H = \tau_A - \tau_D - \tau_R$ (added mass, hydrodynamic damping and restoring forces).

τ_{CS} : forces and motions caused by the control surfaces (rudders, blades etc.).

τ_P: forces and moments generated by the propulsion systems (forces produced by the propellers and thrusters).

τ_{CE}: forces and motions which act on the hull due to environmental disturbances (waves, winds and currents).

Moreover, if the kinematics of the ship is to be included, the following equations are normally used as the vectorial expression of the 6-DOF motion equations:

$$\boldsymbol{M}\dot{\boldsymbol{v}} + \boldsymbol{C}(\boldsymbol{v})\boldsymbol{v} + \boldsymbol{D}(\boldsymbol{v})\boldsymbol{v} + \boldsymbol{g}(\boldsymbol{\eta}) = \boldsymbol{\tau} \quad \textbf{(8)}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{v} \quad \textbf{(9)}$$

where $\boldsymbol{M}=\boldsymbol{M}_{RG}+\boldsymbol{M}_A$ is the inertial matrix (including added mass matrix $\boldsymbol{M}_A$), $\boldsymbol{C}(\boldsymbol{v}) = \boldsymbol{C}_{RB}(\boldsymbol{v}) + \boldsymbol{C}_A(\boldsymbol{v})$ is the matrix of Coriolis and centripetal terms (including added mass matrix $\boldsymbol{C}_A(\boldsymbol{v})$), $\boldsymbol{D}(\boldsymbol{v})$, is the damping matrix, $\boldsymbol{g}_{(\eta)}$ is the vector of gravitational forces and moments and $\boldsymbol{\tau} = \boldsymbol{\tau}_{CS} + \boldsymbol{\tau}_P + \boldsymbol{\tau}_E$ is the vector of the propulsion forces and moments, control surfaces and environmental disturbances.

The concept of added mass is usually misunderstood to be a finite amount of water connected to the vehicle such that the vehicle and the fluid represent a new system with mass larger than the original system. Added (virtual) mass should be understood as pressure-induced forces and moments due to a forced harmonic motion of the body, which is proportional to the acceleration of the body (Fossen, 1994).

The term $\boldsymbol{J}_{(\eta)}$ of equation 9 is the transformation matrix used to represent the position and orientation vector η in the Earth-fixed frame. Equation 9 can be expressed by:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} c(\psi)c(\theta) & -s(\psi)c(\phi)+c(\psi)s(\theta)s(\phi) & s(\psi)s(\phi)+c(\psi)c(\phi)s(\theta) & & & \\ s(\psi)c(\theta) & c(\psi)c(\phi)+s(\phi)s(\theta)s(\psi) & -c(\psi)s(\phi)+s(\theta)s(\psi)c(\phi) & & \mathbf{0}_{3\times3} & \\ -s(\theta) & c(\theta)s(\phi) & c(\theta)c(\phi) & & & \\ & & & 1 & s(\phi)t(\theta) & c(\phi)t(\theta) \\ & \mathbf{0}_{3\times3} & & 0 & c(\phi) & -s(\phi) \\ & & & 0 & s(\phi)/c(\theta) & c(\phi)/c(\theta) \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix} \quad \textbf{(10)}$$

where $s(\cdot) = \sin(\cdot)$, $c(\cdot) = \cos(\cdot)$, $t(\cdot) = \tan(\cdot)$.

HYDRODYNAMIC FORCES AND MOTIONS

An important step in the development of maneuvering models is to expand the forces and moments in Taylor's series. In this way, the hydrodynamic forces and motions are normally represented as a non-linear function of the accelerations $\dot{\nu}$, velocities ν, and Euler angles included in η:

$$\boldsymbol{\tau}_H = \mathbf{f}(\dot{\boldsymbol{\nu}}, \boldsymbol{\nu}, \boldsymbol{\eta}) \tag{11}$$

where the function **f** is calculated through the development in Taylor series of the functions *X*, *Y*, *Z*, *K*, *M* and *N* as, for example, for force *X*:

$$\frac{dX}{dt} = \frac{\partial X}{\partial u}u + \frac{\partial X}{\partial v}v + \ldots + \frac{\partial X}{\partial \dot{u}}\dot{u} + \frac{\partial X}{\partial \dot{v}}\dot{v} + \ldots \frac{1}{2}\frac{\partial^2 X}{\partial u^2}u^2 + \ldots \frac{1}{6}\frac{\partial^3 X}{\partial u^3}u^3 + \ldots \tag{12}$$

and the partial derivatives of the development, termed *hydrodynamic derivatives* or *hydrodynamic coefficients*, are represented by terms such as:

$$X_{\dot{u}} = \frac{\partial X}{\partial \dot{u}}, \quad X_{uu} = \frac{1}{2}\frac{\partial^2 X}{\partial u^2}, \quad Y_{vv} = \frac{1}{2}\frac{\partial^2 Y}{\partial \delta^2}, \quad N_{v|r|} = \frac{\partial^2 N}{\partial v \partial |r|} \text{ y } K_{ppp} = \frac{1}{6}\frac{\partial^3 K}{\partial p^3}, \tag{13}$$

evaluated at equilibrium conditions. The initial condition of motion equilibrium is chosen as straight ahead motion at constant speed (Abkowitz, 1964).

An approximation to the above expressions of equation (11), in stationary state, using the Taylor development around the state of equilibrium $u = u_0$ and $\nu = \dot{\nu} = 0_{6x1}$, obtaining a polynomial of the fourth order or lower (Kallstrom, 1982) if there is lateral symmetry of the ship (Lewis, 1989). In (Abkowitz, 1964) and (Lewis, 1989) it is proposed that up to the third order of development should be taken. No terms higher than the third order are included since experience has shown that their inclusion does not significantly increase accuracy. Moreover, several terms can be discarded due to the lateral symmetry of ships and by taking into account only terms with acceleration of the first order

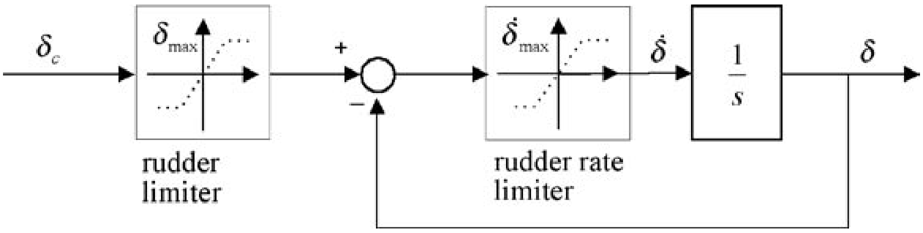
The hydrodynamic derivatives can be determined approximately from the hydrodynamics theory (strip theory) (Lewis, 1989), by experiments using scale models (Lewis, 1989), (Son and Nomoto, 1981), (Blanke and Jensen, 1997) or by system identification methods carrying out experiments on the ships (Astrom and Kallstrom, 1976), (Kallstrom and Astrom, 1981). However, it is difficult to determine all of the hydrodynamic coefficients of a ship. To obtain a good model of the vessel, these coefficients need to be determined reasonably accurately.

RUDDER SATURATION AND DYNAMICS

To include the action of the rudder in the model, the simplified model proposed by Van Amerongen (1982) has been used to represent the steering machine, as shown in Figure 4, where δ_c is the commanded rudder angle and δ is the actual rudder angle.

The rudder angle and rudder rate limiters will typically be in the ranges of $-35^{o} \leq \delta_{max} \leq 35^{o}$ and $2.5^{o}/\sec \leq \dot{\delta}_{max} \leq 7^{o}/\sec$.

Figure 2 Simplified diagram of the rudder control loop



Source: (Van Amerongen, 1982)

DYNAMIC EQUATIONS WITH FOUR DEGREES OF FREEDOM

In some vessels such as container ships, warships or high-speed ferries, as well as the motions of sway, yaw and surge, the roll motion must also be included in the mathematical model. If the coordinate system O_B is located in the ship to coincide with the main inertial axes, equation (4) shortens to:

$$\boldsymbol{M}_{RB(4x4)}\,\dot{\boldsymbol{v}} + \boldsymbol{C}_{RB(4x4)}(\boldsymbol{v})\,\boldsymbol{v} = \boldsymbol{\tau}_{RB(4x1)} \tag{14}$$

where the velocity vector is given by:

$$\boldsymbol{v} = [u,\ v,\ p,\ r]^{T} \tag{15}$$

The mass and inertia matrix has the form:

$$\boldsymbol{M}_{RB(4x4)} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & -mz_G & mx_G \\ 0 & -mz_G & I_x & 0 \\ 0 & mx_G & 0 & I_z \end{bmatrix} \tag{16}$$

The centripetal and Coriolis force is:

$$\boldsymbol{C}_{RB(4x4)}(\boldsymbol{v}) = \begin{bmatrix} 0 & 0 & mz_G r & -m(x_G r + v) \\ 0 & 0 & 0 & mu \\ -mz_G r & o & 0 & 0 \\ m(x_G r + v) & -mu & 0 & 0 \end{bmatrix} \tag{17}$$

The vector of the total external forces at eh axes x_B and y_B (X, Y) and the total external motions at the axes z_B and y_B (N y K) are:

$$\boldsymbol{\tau}_{RB(4x1)} = [X,\ Y,\ K,\ N]^{T} \tag{18}$$

which can be more concisely represented as:

$$\begin{aligned} &\text{Surge:} \quad m(\dot{u} - vr - x_G r^2 + z_G pr) = X \\ &\text{Sway:} \quad m(\dot{v} - ur - z_G \dot{p} + x_G \dot{r}) = Y \\ &\text{Roll:} \quad I_x \dot{p} - m z_G (ur + \dot{v}) = K \\ &\text{Yaw:} \quad I_z \dot{r} + m x_G (ur + \dot{v}) = N \end{aligned} \tag{19}$$

MODELS WITH 4 DEGREES OF FREEDOM

This section reviews some models, presented by various authors, which can be used to simulate the dynamic behaviour of a ship. These models include hydrodynamic forces and motions

Son and Nomoto Model

Son and Nomoto presented a non-linear mathematical model of a high-speed container ship (Son and Nomoto, 1981) in order to study the coupled motions of yaw, sway and roll. In this model, the basic motion equations were formulated with four degrees of freedom of equation (19) as follows:

$$\begin{aligned} (m+m_x)\dot{u} - (m+m_y)vr &= X \\ (m+m_y)\dot{v} - (m+m_x)ur + m_y\alpha_y\dot{r} - m_y l_y \dot{p} &= Y \\ (I_z + J_z)\dot{r} + m_y\alpha_y\dot{v} &= N - Yx_G \\ (I_x + J_x)\dot{p} - m_y I_y \dot{v} - m_x l_x ur + WGM\phi &= K_0 \end{aligned} \tag{20}$$

where m_x y m_y are added mass in the surge and yaw directions, J_x and J_z the added inertia about roll and axes respectively. The centre of added mass for m_y is denoted by α_y (x-coordinate), while lx and ly are the added mass coordinates of m_x and m_y respectively.

The origin of the fixed coordinates of the ship is described as $[x_G, 0, 0]$. The terms for added mass and inertias are expressly included with their corresponding radius of rotation rather than including them by means of their hydrodynamic coefficients, as was the case in equation (19). The metacentric restoring moment in roll was added in the K expression, where W is the weight of the water displaced by the ship hull and GM is the transverse metacentric height. The term x_G appears in the equation since the hydrodynamic motion of yaw N is defined around the geometrical centre of the ship. The hydrodynamic forces and motions are developed using the hydrodynamic derivatives of equations (20).

$$= X(u) + (1-t)T(J) + X_{vr}\,vr + X_{vv}v^2 + + X_{rr}\,r^2 + X_{\varphi\varphi}\varphi^2 + c_{RX}\,F_N \operatorname{sen}\delta \tag{21}$$

$$\begin{aligned} Y = {} & Y_v\,v + Y_r\,r + Y_{\dot\phi}\dot{\phi} + Y_\phi\phi + Y_{vvv}\,v^3 + Y_{rrr}r^3 + Y_{vvr}v^2 r + Y_{vrr}vr^2 + \\ & + Y_{vv\phi}v^2\phi + Y_{v\phi\phi}v\phi^2 + Y_{rr\phi}r^2\phi + Y_{r\phi\phi}r\phi^2 + (1+a_H)F_N\cos\delta \end{aligned} \tag{22}$$

$$N = N_v v + N_r r + N_{\dot{\phi}} \dot{\phi} + N_{\phi} \phi + N_{vvv} v^3 + N_{rrr} r^3 + N_{vvr} v^2 r + N_{vrr} v r^2 + \\ + N_{vv\phi} v^2 \phi + N_{v\phi\phi} v \phi^2 + N_{rr\phi} r^2 \phi + N_{r\phi\phi} r \phi^2 + (x_R + a_H x_H) F_N \cos\delta \quad \textbf{(23)}$$

$$K = K_v v + K_r r + K_{\dot{\phi}} \dot{\phi} + K_{\phi} \phi + K_{vvv} v^3 + K_{rrr} r^3 + K_{vvr} v^2 r + K_{vrr} v r^2 + \\ + K_{vv\phi} v^2 \phi + K_{v\phi\phi} v \phi^2 + K_{rr\phi} r^2 \phi + K_{r\phi\phi} r \phi^2 - (1 + a_H) z_R F_N \cos\delta \quad \textbf{(24)}$$

The forces produced by the rudder are represented by the last terms of each equation. The force of the rudder F_N is expressed as:

$$F_N = -\frac{6{,}13\Lambda}{\Lambda + 2{,}25} \cdot \frac{A_R}{L^2} \left(u_R^2 + v_R^2\right) sen\alpha_R \quad \textbf{(25)}$$

where α_R is the incidence flow angle, u_R and v_R are the surge and sway components of the incident flow velocity in the rudder defined by equations 26, 27 and 28, respectively:

$$\alpha_R = \delta + \arctan\left(v_R / u_R\right) \quad \textbf{(26)}$$

$$v_R = \gamma v + c_{Rr} r + c_{Rrrr} r^3 + c_{Rrrv} r^2 v \quad \textbf{(27)}$$

$$u_R = u_p \varepsilon \sqrt{1 + 8kK_T / (\pi J^2)} \quad \textbf{(28)}$$

$$J = u_p V / (nD) \quad \textbf{(29)}$$

$$u_p = \cos v \left[(1 - w_p) + r \left\{ (v + x_p r)^2 + c_{pv} v + c_{pr} r \right\} \right] \quad \textbf{(30)}$$

Table 2 shows the main features of the ship identified by Son and Nomoto in their research.

Table 2: Container Ship SR 108 Main Features

Description		Symbol	Value	units
Length		L	175,00	m
Breadth		B	25,40	m
Draft	fore	d_F	8	m
	aft	d_A	9	m
	mean	d	8,5	m
Displacement volume		∇	21.222	m^3
Height from keel to transverse metacenter		KM	10,39	m
Height from keel to centre of buoyancy		KB	4,6154	m
Block coefficient		C_B	0,559	
Prismatic coefficient		C_p	0,580	
Rudder Area		A_R	33,0376	m^2
Aspect Ratio		Λ	1,8219	
Propeller Diameter		D	6,533	m

Table 3 shows the hydrodynamic coefficients and other parameters of the model

Table 3: Hydrodynamic derivatives and coefficients

a) Only hull parameters

m	0,00792	*Yp*	0,0	*Nvvφ*		-0,019058
mx	0,000238	*Yφ*	-0,000063	*Nvφφ*		-0,0053766
my	0,007049	*Yvvv*	-0,109	*Nrrφ*		-0,0038592
Ix	0,0000176	*Yrrr*	0,00177	*Nrφφ*		0,0024195
Jx	0,0000034	*Yrvv*	0,0214	*Kv*		0,0003026
Iz	0,000456	*Yrrv*	-0,0405	*Kr*		-0,0003026
Jz	0,000419	*Yvvφ*	0,04605	κφ	0,1	($Fn \leq 0,1$)
αy	0,05	*Yvφφ*	0,0034		0,2	($Fn \geq 0,2$)
lx	0,0313	*Yrrφ*	0,009325		*Fn*	($0,1<Fn<0,2$)
ly	0,0313	*Yrφφ*	-0,001368	*Kφ*		-0,000021
KT	0,527-0,455J	*Nv*	-0,0038545	*Kvvv*		0,002843
Xuu	-0,0004226	*Nr*	-0,00222	*Krrr*		-0,0000462
Xuu	-0,00311	*Np*	0,000213	*Krvv*		-000558
Xuu	0,00386	*Nφ*	-0,0001424	*Krrv*		0,0010565
Xuu	0,00020	*Nvvv*	0,001492	*Kvvφ*		-0,0012012
Xuu	-0,00020	*Nrrr*	-0,00229	*Kvφφ*		-0,0000793
Yv	-0,0116	*Nrvv*	-0,0424	*Krrφ*		-0,000243
Yr	0,00242	*Nrrv*	0,00156	*Krφφ*		0,00003569

b) Propeller and rudder parameters

Np	79,10 (Fn 0,2)	*αH*	0,237	ε	0,921
(*rpm*)	118,64 (Fn 0,3)	*xH*	-0,48	*k*	0,631
	158,19 (Fn 0,4)	*cRX*	0,71	γ	0,088 ($v > 0$)
(1- *t*)	0,825	*zR*	0,033		0,193 ($v \leq 0$)
(1-ω*p*)	0,816	*cpv*	0,0	*cRr*	-0,156
xR	-0,5	*cpr*	0,0	*cRrrr*	-0,275
xp	-0,526	τ	1,09	*cRrrv*	1,96

Motion equation (19) can be described as:

$$\begin{bmatrix} (m+m_x) & 0 & 0 & 0 \\ 0 & (m+m_y) & m_y\alpha_y & -m_y l_y \\ 0 & -m_y I_y & (I_x+J_x) & 0 \\ 0 & m_y\alpha_y & 0 & (I_z+J_z) \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} X \\ Y \\ K \\ N \end{bmatrix} + \begin{bmatrix} (m+m_y)vr \\ (m+m_x)ur \\ m_x l_x ur - WGM\phi \\ -Yx_G \end{bmatrix} \quad \textbf{(31)}$$

Blanke and Jensen's Model

Blanke and Jensen presented a non-linear mathematical model of a high-speed container ship (Blanke and Jensen, 1997) obtained using a roll planar motion mechanism (RPMM) with four degrees of freedom. The model was developed in order to predict the manoeuvring characteristics with more accuracy than that obtained previously in order to study the coupled motions of yaw, sway and roll. The basic equations of motion with four degrees of freedom of equation (19) were formulated as follows:

$$\begin{aligned} & m(\dot{u} - vr - x_G r^2 + z_G pr\cos\phi) = X \\ & m(\dot{v} - ur - z_G \dot{p}\cos\phi + x_G \dot{r}) = Y \\ & I_x \dot{p} - m z_G (ur + \dot{v})\cos\phi = K - \rho g \nabla G_z(\phi) \\ & I_z \dot{r} + m x_G (ur + \dot{v}) = N \end{aligned} \quad \textbf{(32)}$$

where ∇ indicates the displacement of the ship, g the gravity constant, δ the density of the water, I_x and I_z are the inertia motions of the ship with respect to the axes x_B and z_B, *m* is the ship's mass and the centre of gravity is assumed to be in the position $(x_G, 0, z_G)$.

The results of the experiments with the RPMM were formulated with a full adimensional hydrodynamic model using the prime system:

$$U = \sqrt{(u_0 + u)^2 + v^2} \tag{33}$$

In the RPMM model, the relative velocity of adimensional advance in hydrodynamic terms is used:

$$u'_{\mathrm{a}} = \frac{U - U_{norm}}{U} \tag{34}$$

In the reference, it is noticeable that u'_{a} is different from the relative velocity of adimensional advance *u'=u/U* which is included when the accelerations of equation (32) are calculated. The terms X, Y, N and K of the hydrodynamic forces were:

$$\begin{aligned} X' = {} & X'_0 + X'_u u'_{\mathrm{a}} + X'_{uu} u'^2_{\mathrm{a}} + X'_{uuu} u'^3_{\mathrm{a}} + X'_{vr} v'r' + X'_{rr} r'^2 + X'_\delta \delta' + X'_{\delta\delta} \delta'^2 \\ & + X'_{\delta u} \delta' u'_{\mathrm{a}} + X'_{\delta\delta u} \delta'^2 + X'_{\delta uu} \delta' u'^2_{\mathrm{a}} + X'_v v' + X'_{vv} v'^2 + X'_{\delta v} \delta' v' + X'_{\delta vv} \delta' v'^2 \\ & + X'_{v\phi} v'\phi' + X'_{v\phi\phi} v'\phi'^2 + X'_\phi \phi' + X'_{\phi\phi} \phi'^2 + X'_{\phi vv} \phi' v'^2 + X'_r r' + X'_{pp} p'^2 \\ & + X'_{ppu} p'^2 u'^2_{\mathrm{a}} + X'_{ext} \end{aligned} \tag{35}$$

$$\begin{aligned} Y' = {} & Y'_0 + Y'_{0u} u'_{\mathrm{a}} + Y'_{\dot v} \dot v' + Y'_{\dot r} \dot r' + Y'_{\dot p} \dot p' + Y'_v v' + Y'_{vv} v'^2 + Y'_{v|v|} v'|v'| + Y'_\delta \delta' \\ & + Y'_{\delta\delta} \delta'^2 + Y'_{\delta\delta\delta} \delta'^3 + Y'_{\delta u} \delta' u'_{\mathrm{a}} + Y_{\delta\delta u} \delta'^2 u'_{\mathrm{a}} + Y_u u'_{\mathrm{a}} + Y'_{uu} u'^2_{\mathrm{a}} + Y'_{uuu} u'^3_{\mathrm{a}} + Y'_{\delta u} \delta' u'_{\mathrm{a}} \\ & + Y'_{\delta\delta u} \delta'^2 u' + Y'_{\delta\delta\delta u} \delta'^3 u'_{\mathrm{a}} + Y'_{\delta v} v'\delta' + Y'_{\delta vv} \delta' v'^2 + Y'_\phi \phi' + Y'_{\phi\phi} \phi'^2 + Y'_{v\phi} v'\phi' \\ & + Y'_{v\phi\phi} v'\phi'^2 + Y'_{\phi vv} \phi' v'^2 + Y'_r r' + Y'_{rrr} r'^3 + Y'_{r|v|} r'|v'| + Y'_{v|r|} v'|r'| + Y'_{vrr} v'r'^2 \\ & + Y'_p p' + Y'_{p|p|} p'|p'| + Y'_{ppp} p'^3 + Y'_{pu} p'u'_{\mathrm{a}} + Y'_{pu|pu|} p'u'_{\mathrm{a}} |p'u'_{\mathrm{a}}| + Y'_{ext} \end{aligned} \tag{36}$$

$$\begin{aligned} K' = {} & K'_0 + K'_{0u} u'_{\mathrm{a}} + K'_{\dot v} \dot v' + K'_{\dot r} \dot r' + K'_{\dot p} \dot p' + K'_v v' + K'_{vv} v'^2 + K'_{v|v|} v'|v'| + K'_\delta \delta' \\ & + K'_{\delta\delta\delta} \delta'^3 + K'_{\delta u} \delta' u'_{\mathrm{a}} + K'_{\delta\delta u} \delta'^2 u'_{\mathrm{a}} + K'_u u'_{\mathrm{a}} + K'_{uu} u'^2_{\mathrm{a}} + K'_{uuu} u'^3_{\mathrm{a}} + K'_{\delta u} \delta' u'_{\mathrm{a}} \\ & + K'_{\delta\delta u} \delta'^2 u'_{\mathrm{a}} + K'_{\delta\delta\delta u} \delta'^3 u'_{\mathrm{a}} + K'_{\delta v} v'\delta' + K'_{\delta vv} \delta'' v'^2 + K'_\phi \phi' + K_{\phi\phi} \phi'^2 + K'_{v\phi} v'\phi' \\ & + K_{v\phi\phi} v'\phi'^2 + K'_{\phi vv} \phi' v'^2 + K'_r r' + K'_{rrr} r'^3 + K'_{r|v|} r'|v'| + K'_{v|r|} v'|r'| + K'_{vrr} v'r'^2 \\ & + K'_p p' + K'_{p|p|} p'|p'| + K'_{ppp} p'^3 + K'_{pu} p'u'_{\mathrm{a}} + K'_{pu|pu|} p'u'_{\mathrm{a}} |p'u'_{\mathrm{a}}| + K'_{ext} \end{aligned} \tag{37}$$

$$\begin{aligned} N' = {} & N'_0 + N'_{0u} u'_{\mathrm{a}} + N'_{\dot v} \dot v' + N'_{\dot r} \dot r' + N'_{\dot p} \dot p' + N'_v v' + N'_{vv} v'v' + N'_{v|v|} v'|v'| + N'_\delta \delta' \\ & + N'_{\delta\delta} \delta'^2 + N'_{\delta\delta\delta} \delta'^3 + N'_{\delta u} \delta' u'_{\mathrm{a}} + N'_{\delta\delta u} \delta'^2 u'_{\mathrm{a}} + N'_u u'_{\mathrm{a}} + N'_{uu} u'^2_{\mathrm{a}} + N'_{uuu} u'^3_{\mathrm{a}} + N'_{\delta u} \delta u'_{\mathrm{a}} \\ & + N'_{\delta\delta u} \delta'^2 u'_{\mathrm{a}} + N'_{\delta\delta\delta u} \delta'^3 u'_{\mathrm{a}} + N'_{\delta v} \delta' v' + N'_{\delta vv} \delta' v'^2 + N'_\phi \phi' + N'_{\phi\phi} \phi'^2 + N'_{v\phi} v'\phi' \\ & + N'_{v\phi\phi} v'\phi'^2 + N'_{\phi vv} \phi' v'^2 + N'_r r' + N'_{rrr} r'^3 + N'_{r|v|} r'|v'| + N'_{v|r|} v'|r'| + N'_{vrr} v'r'^2 \\ & + N'_{pu} p'u'_{\mathrm{a}} + N'_p p' + N'_{p|p|} p'|p'| + N'_{ppp} p'^3 + N'_{pu|pu|} p'u'_{\mathrm{a}} |p'u'_{\mathrm{a}}| + N'_{ext} \end{aligned} \tag{38}$$

where X'_0 is the force in direction x_B in the equilibrium condition $U = u_0$ and N'_0, K'_0 and Y'_0 the motions N, K and the force Y for $v = r = p = \delta = 0$. The terms represent the effects of wind, waves and currents.

For the design of the controllers, it is difficult to use the non-linear model directly. For this reason, the non-linear model must be replaced by a linear model, since most theorems have been obtained using linear theory. It is easy to obtain a linear model from the non-linear model by eliminating all of the terms whose orders are greater than one in equation (32).

For the study of the motions of roll and yaw the surge equation is not considered due to the small coupling between the motions of yaw, sway and roll with the surge motion. Thus, the linear model has only five states $\mathbf{x} = [v\ p\ r\ \phi\ \psi]^T$ and, if the rudder angle $\mathbf{u} = [\delta]$ is defined as input, the linear model can be expressed as:

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \tag{39}$$

where the matrixes **A** and **B** are defined by:

$$\mathbf{A} = \mathbf{M}^{-1}\mathbf{F} \qquad \mathbf{B} = \mathbf{M}^{-1}\mathbf{G} \tag{40}$$

with

$$\mathbf{F} = \begin{bmatrix} Y'_v + Y'_{uv}\Delta u' & Y'_p + Y'_{up}\Delta u' & Y'_r - m'\Delta u' & Y'_\phi & 0 \\ K'_v + K'_{uv}\Delta u' & K'_p + K'_{up}\Delta u' & K'_r + m'z'_G\Delta u' & K'_\phi - \rho' g' \nabla' GM' & 0 \\ N'_v + N'_{uv}\Delta u' & N'_p + N'_{up}\Delta u' & N'_r + m'x'_G\Delta u' & N'_\phi & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{41}$$

$$\mathbf{G} = \begin{bmatrix} Y'_\delta + Y'_{\delta u}\Delta u' \\ K'_\delta + K'_{\delta u}\Delta u' \\ N'_\delta + N'_{\delta u}\Delta u' \\ 0 \\ 0 \end{bmatrix} \tag{42}$$

CONCLUSIONS

Mathematical models of ships are used both in the simulation and in the design of control systems in applications such as ship steering control or the study of sea vessels which act cooperatively.

A review has been made of several models of four degrees of freedom which take into account the yaw, sway and roll couplings. These models are useful in some types of ships such as warships, container ships, fast ferries, etc. These models obtain a better agreement between the results predicted in simulation and those obtained experimentally.

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APENDICE: MODELOS MATEMATICOS DE BUQUES PARA SIMULACIÓN Y CONTROL DE MANIOBRA

RESUMEN

En este artículo se revisan modelos con cuatro grados de libertad, que incluyen el movimiento de balance, para describir el movimiento de un buque. Así, se tienen en cuenta los acoplamientos de guiñada, abatimiento (desplazamiento lateral) y balance. Estos modelos se utilizan en aplicaciones como la simulación y control del gobierno de un buque, o el estudio de vehículos marinos que actúan cooperativamente.

Palabras clave: Modelos matemáticos. Movimiento del buque. Maniobra. Simulación.

INTRODUCCIÓN

El conocimiento de las dinámicas relacionadas con la guiñada, el desplazamiento lateral del buque y el balance, es útil tanto para mejorar los modelos de maniobra como también, por ejemplo, esencial para desarrollar aplicaciones de control de amortiguamiento de balance en las que los acoplamientos dinámicos entre dichos movimientos son muy importantes. En este artículo se describen modelos de buques de cuatro grados de libertad que se pueden utilizar para la simulación del comportamiento de maniobra y en aplicaciones de control. Se han incluido los modelos hidrodinámicos de dos buques contenedores (Son and Nomoto, 1981) and (Blanke and Jensen, 1997) obtenidos utilizando modelos cautivos en un canal de ensayos hidrodinámicos que incluyen el movimiento de balance.

METODOLOGÍA: MODELOS MATEMÁTICOS DE BUQUES

En primer lugar se presenta la notación estándar utilizada en la descripción del movimiento de vehículos marinos en seis grados de libertad (GDL) y que se muestran en la tabla 1.

Tabla 1: Notación y descripción de los GDL

GDL	Traslaciones	Fuerzas	Velocidades lineales	Posiciones
1	avance	X	u	x
2	abatimiento	Y	v	y
3	arfada	Z	w	z
	Rotaciones	**Momentos**	**Velocidades angulares**	**Ángulos**
4	balanceo	K	p	ϕ
5	cabeceo	M	q	θ
6	guiñada	N	r	ψ

Así, el movimiento de un buque en seis GDL se puede describir con los vectores:

$$\mathbf{v}=[u,\ v,\ w,\ p,\ q,\ r]^T,\quad \boldsymbol{\tau}=[X,\ Y,\ Z,\ K,\ M,\ N]^T,\quad \boldsymbol{\eta}=[x,\ y,\ z,\ \phi,\ \theta,\ \psi]^T \qquad (1)$$

y las ecuaciones del movimiento que se definen de forma vectorial por las ecuaciones:

$$\boldsymbol{M}\,\dot{\boldsymbol{v}}+\boldsymbol{C}(\boldsymbol{v})\boldsymbol{v}+\boldsymbol{D}(\boldsymbol{v})\boldsymbol{v}+\boldsymbol{g}(\boldsymbol{\eta})=\boldsymbol{\tau}$$
$$\dot{\boldsymbol{\eta}}=\boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{v} \qquad (2)$$

donde M es la matriz de inercia y $C(v)$ la matriz de Coriolis y términos centrípetos (incluyendo en ambas las matrices de masas añadidas). ***D(v)*** es la matriz de amortiguamiento, $\boldsymbol{g}(\eta)$ un vector de fuerzas y momentos debidos a la gravedad y el empuje y τ un vector de las fuerzas y momentos de la propulsión, superficies de control y perturbaciones ambientales. Las masas añadidas (virtuales) deben entenderse como las fuerzas inducidas de presión y los momentos debidos a un movimiento armónico forzado del casco que es proporcional a su aceleración. El término $\boldsymbol{J}(\eta)$ es la matriz de transformación utilizada para representar el vector η de posición y orientación.

Las fuerzas y momentos hidrodinámicos se suelen representar como una función no lineal de las aceleraciones $\dot{\nu}$, velocidades ν, y los ángulos de Euler incluidos en η:

$$\boldsymbol{\tau}_H=\mathbf{f}(\dot{\mathbf{v}},\ \mathbf{v},\ \boldsymbol{\eta}) \qquad (3)$$

donde la función **f** se calcula mediante el desarrollo de Taylor de las funciones *X*, *Y*, *Z*, *K*, *M* y *N* y las derivadas parciales del desarrollo, denominadas *derivadas* o *coeficientes hidrodinámicos*, están evaluadas con el buque navegando en avante a una velocidad constante ($u = u_0$ and $\nu = \dot{\nu} = 0_{6x1}$).

Las derivadas hidrodinámicas pueden determinarse de forma aproximada a partir de la teoría hidrodinámica, por experimentos con modelos a escala o mediante identificación de sistemas realizando experimentos en los buques. Para obtener un buen modelo del buque hay que determinarlos con una exactitud razonable. Para incluir en el modelo la acción del timón, se ha utilizado el modelo simplificado sugerido por Van Amerongen.

En algunos buques como por ejemplo buques contenedores, de guerra o en los ferries de alta velocidad, se debe incluir también el movimiento de balance en el modelo matemático. La ecuación del movimiento en cuatro grados de libertad es:

$$\begin{aligned}
&\text{Avance:} && m(\dot{u}-vr-x_G r^2+z_G pr)=X\\
&\text{Desp. lateral:} && m(\dot{v}-ur-z_G\dot{p}+x_G\dot{r})=Y\\
&\text{Balanceo:} && I_x\dot{p}-mz_G(ur+\dot{v})=K\\
&\text{Guiñada:} && I_z\dot{r}+mx_G(ur+\dot{v})=N
\end{aligned} \qquad (4)$$

En este trabajo se realiza una revisión de algunos de los modelos, que se pueden utilizar para simular el comportamiento dinámico de un buque. En primer lugar se presenta el *modelo de Son y Nomoto* expresado por la ecuación (5) que es un modelo matemático no lineal con cuatro GDL. El modelo incluye el desarrollo de las fuerzas y momentos hidrodinámicos, la fuerza del timón, las principales características del buque y los valores de los coeficientes hidrodinámicos y otros parámetros del modelo.

$$\begin{aligned} (m+m_x)\dot{u}-(m+m_y)vr &= X \\ (m+m_y)\dot{v}-(m+m_x)ur+m_y\alpha_y\dot{r}-m_yl_y\dot{p} &= Y \\ (I_z+J_z)\dot{r}+m_y\alpha_y\dot{v} &= N-Yx_G \\ (I_x+J_x)\dot{p}-m_yl_y\dot{v}-m_xl_xur+WGM\phi &= K_0 \end{aligned} \quad \textbf{(5)}$$

El segundo modelo presentado corresponde al *modelo de Blanke y Jensen* que corresponde al modelo matemático no lineal de un buque contenedor de alta velocidad. El modelo se desarrolló con el objeto de predecir las características de maniobra con una mayor precisión que la obtenida anteriormente, con el fin de investigar los movimientos acoplados de guiñada, desplazamiento lateral y balance. Las ecuaciones básicas del movimiento con cuatro GDL de la ecuación (4) las formularon como sigue:

$$\begin{aligned} & m(\dot{u}-vr-x_Gr^2+z_Gpr\cos\phi)=X \\ & m(\dot{v}-ur-z_G\dot{p}\cos\phi+x_G\dot{r})=Y \\ & I_x\dot{p}-mz_G(ur+\dot{v})\cos\phi=K-\rho g\nabla G_z(\phi) \\ & I_z\dot{r}+mx_G(ur+\dot{v})=N \end{aligned} \quad \textbf{(6)}$$

Para el diseño de controladores es difícil utilizar directamente el modelo no lineal. Por esta razón, debe reemplazarse por un modelo lineal. Es fácil obtener un modelo lineal a partir del modelo no lineal eliminando todos los términos cuyos órdenes sean mayores que uno en la ecuación (6). Para el estudio de los movimientos de balance y guiñada, la ecuación de avance no se considera debido al pequeño acoplamiento entre los movimientos de guiñada, desplazamiento lateral y balance con el movimiento de avance. Por lo tanto, el modelo lineal tiene sólo cinco estados $[v\ p\ r\ \phi\ \psi]^T$

CONCLUSIONES

Los modelos matemáticos de buques se emplean tanto en simulación como en el diseño de sistemas de control en aplicaciones tales como el control del gobierno de un buque, o el estudio de vehículos marinos que actúan cooperativamente.

Se ha efectuado una revisión de diferentes modelos de cuatro grados de libertad, que tienen en cuenta los acoplamientos de guiñada, desplazamiento lateral y balance. Estos modelos están indicados en algunos tipos de buques tales como buques de guerra, contenedores, ferries de alta velocidad, etc. Con ellos se obtiene una mejor adecuación entre los resultados previstos en simulación, y los obtenidos experimentalmente.