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Application of Random Forest in the Prediction of Extreme Flows: Evaluation against Deterministic Methods, exper-imental case of the Magdalena River Frío basin in Colombia

Sully M. Quintero S. [1,2,*], Andres F. Hatum P. [1,3], Jolahuz A. Quintero S.[1]

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ABSTRACT

Hydrological and hydraulic modeling is essential in assessing and managing water resources, especially in regions prone to extreme events. The Frío River basin, located in the department of Magdalena, faces recurrent floods and water deficits, impacting socio-environmental resilience and territorial planning. This study uses advanced hydrological and hydraulic modeling tools, such as HEC-RAS and hydrometeorological series analysis, to evaluate river dynamics under extreme precipitation scenarios and changes in land use. The Random Forest (RF) algorithm was implemented as a machine learning model to improve predictive capacity, using historical precipitation data to estimate flows in extreme events with greater precision. The model was trained with historical records, allowing the identification of non-linear relationships between hydrometeorological variables and the hydrological response of the basin. The channel's current state was analyzed, considering historical precipitation data and projected scenarios with different urbanization rates and changes in vegetation cover. Climate prediction models were incorporated to assess the impact of climate change on the recurrence of extreme events and water availability. Using Random Forest as a hydrological modeling tool improved the estimation of extreme flows, providing a scientific basis for water risk management and formulating adaptation strategies. Finally, methods are proposed to strengthen the resilience of the basin to climate variability and to optimize water management through ecological restoration and sustainable territorial planning.

1. Introduction.

There are few studies on the behavior of the Río Frío basin. However, one study established a correlation between the characteristics of the morphometric parameters and the surface drainage variable of the basins that flow into the Ciénaga Grande del Magdalena [1]. The objective was to characterize the hydrographic basin formed by the rivers that originate and flow along the southern and southwestern slopes of the Sierra Nevada de Santa Marta, performing morphometric and drainage analyses to establish correlations with the average surface runoff flow. Key factors such as soil types and slopes were identified, which allowed the basin and its 22 sub-basins to be delimited. Among its main results, it was determined that 8.7% of the sub-basins had streams with sinuous alignments, while 91.3% had straight streams. In addition, it was concluded that most of the drainage types were detrital and of order 3.

CORPAMAG also analyzed the supply and demand of water resources in the Río Frío irrigation district using flow measurements. [2] Their research identified that the river's average flow was approximately 10 m^3/s, decreasing significantly in the summer months. A relevant finding of this study was the

[1]Universidad del Magdalena, Santa Marta, Colombia; squintero@unimagdalena.edu.co (SMQS); ahatum@unimagdalena.edu.co (AFHP); jolahuzquin-teroas@unimagdalena.edu.co (JAQS).

[2]Departamento de Ingeniería Hidráulica y Ambiental, Escuela de Ingeniería, Pontificia Universidad Católica de Chile, Santiago, Chile; squinteros9@estudiante.uc.cl.

[3]Faculty of Engineering, Architecture and Urbanism and Geography, the Federal University of Mato Grosso do Sul, Campo Grande 79070-900, Brazil.

*Corresponding author: Sully M. Quintero S. E-mail Address: squintero@unimagdalena.edu.co.

concession of approximately 5.1 m^3/s to the irrigation districts, which showed overexploitation of the tributary at certain times of the year.

In a later study, hydrological models were applied to evaluate the behavior of the Frío River under extreme scenarios. [3]Their research highlighted the influence of land use on the river dynamics of the basin, indicating that activities such as livestock farming have generated changes in the morphology of the channel. Likewise, the presence of high flows in certain sections, exceeding the edges of the river, and the increase in speeds in some sectors, which increase the probability of erosion, were identified.

There is also an analysis of climate variability and compliance with the Hydrographic Basin Management and Planning Plan (POMCA) [4], which showed the lack of implementation of management strategies despite a structured plan. In addition, a significant limitation was identified in the availability of information, resulting from the absence of climatological equipment at strategic points and adequate hydrological simulation models.

Given these problems, hydrological and hydraulic modeling of the Río Frío is an essential tool for water resource management and natural disaster mitigation. The Río Frío basin is a complex hydrological system that covers a large area from the western slope of the Sierra Nevada to the Ciénaga Grande de Santa Marta, including the Río Frío district and the municipalities of Ciénaga and the Zona Bananera. This river plays a fundamental role in water supply for human consumption in rural and urban areas and in the irrigation of agricultural plantations, particularly banana plantations, which are essential for the local economy. In addition, its flow is crucial for the maintenance of the aquatic ecosystems of the Ciénaga Grande, one of the most important wetlands in the country. Understanding and modeling its behavior is essential for water security, the well-being of communities, and the protection of biodiversity in the region.

In recent years, hydrological modeling has evolved with machine learning techniques that allow for improved accuracy in flow estimation and the evaluation of extreme hydrological scenarios. One of the most prominent approaches in this field is the Random Forest algorithm, which has demonstrated high effectiveness in predicting flows by integrating multiple climatic and hydrological variables. This machine learning model can handle non-linear and complex relationships in the data, allowing for more accurate predictions than traditional methods.

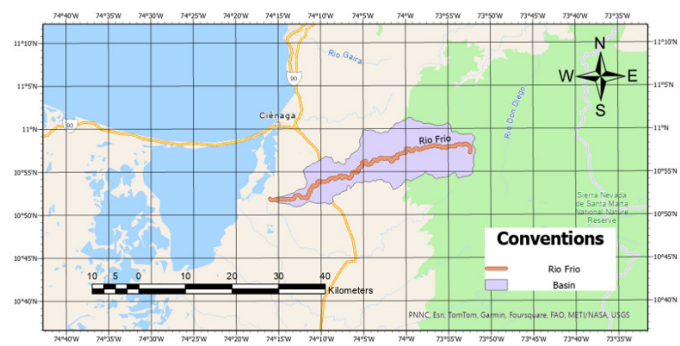
Studies have shown that using Random Forests in hydrological modeling significantly improves the prediction capacity of extreme events [5] since the model can identify patterns in large volumes of data and reduce overfitting by dividing the analysis into multiple decision trees. In this context, the present research implements Random Forest to optimize the estimation of flows in the Frío River basin, comparing it with traditional methodologies and evaluating its effectiveness in modeling extreme hydrological events.

2. Materials and Methods.

2.1. Study area.

The Frío River region is located between the municipalities of Ciénaga and Zona Bananera, in the department of Magdalena. Its source is located on the western slope of the Sierra Nevada de Santa Marta, encompassing the coordinates 10°50'N, 74°16'W and 11°1'N, 73°51'W, and flowing into the Ciénaga Grande de Santa Marta, in a section delimited by the coordinates 10°52'6.28"N, 74°19'17.98"W and 10°52'7.21"N, 74° 19'26.85"W. The basin is bordered to the north by the Córdoba and Guachaca Rivers, to the east by the Buritaca and Don Diego Rivers, to the south by the Sevilla and Orihuela Rivers, and to the west by the La Ciénaga del Chino basin, an integral part of the Ciénaga Grande de Santa Marta complex [6]. The hydrographic network of the Frío River includes several tributaries.

Figure 1: Rio Frio Basin.



Source: Authors.

2.2. Data collection and processing.

The research was developed using a descriptive approach and a case study, combining qualitative and quantitative analysis. A review of secondary information was carried out to characterize the Frío River basin and its hydro-environmental context. Subsequently, a quantitative approach was used to assess the basin's needs for sustainable development.

Data on the water ecosystem, precipitation, and demographics were collected from the databases of the Autonomous Corporation of Magdalena (CORPAMAG), the Institute of Hydrology, Meteorology and Environmental Studies (IDEAM), and the National Administrative Department of Statistics (DANE) to assess surface and underground water supply. [6]The hydrogeological provinces to which the basin belongs were identified based on the classification proposed in [7], and records of water extraction from wells and cisterns were analyzed, considering their impact on resource availability. Additionally, the water use and aridity indices were reviewed, referencing the methodology formulated by [8].

Data from the DANE 2020 National Population and Housing Census were used for demographic characterization. [9], which allowed for determining the population distribution and growth in the study area. The geometric method, which assumes a constant growth rate at regular intervals, was applied for the population projection. [10,11]. The estimate of future water demand was based on the allocation established in Resolution 0779 of 2021 of the Ministry of Housing, City, and Territory of Colombia, which defines an average consumption of 140 liters per inhabitant per day.

2.2.1. Data processing and analysis.

The information obtained delimited critical areas prone to landslides, erosion, and flooding using ArcGIS. This analysis allowed defining natural drainage courses and calculating key morphometric parameters, such as shape factor, compactness coefficient, slope, sinuosity, and drainage density. These indicators are essential to characterizing the basin's hydrological dynamics and susceptibility to erosion and runoff processes.

To estimate extreme flows, intensity - duration - frequency (IDF) curves were generated from historical IDEAM precipitation records between 2000 and 2019, obtained from the IDEAM El Enano (code 29060160) and San Isidro (code 29060280) stations, both located in the municipality of Zona Bananera. Correlations were established with climatic variables, such as maximum precipitation in 24 hours and the height of the meteorological stations, following the recommendations of the drainage manual of the National Roads Institute of Colombia. [12].

In hydrodynamic modeling, Manning's n roughness coefficient was determined by field reconnaissance, considering factors such as surface roughness, vegetation, channel irregularity, and sedimentation. The roughness values were selected based on the tables proposed in[13]

The Random Forest model was implemented to predict flow rates in extreme events, training it with historical precipitation data and hydrometeorological records from IDEAM. The modeling allowed the identification of patterns in the relationship between intense rainfall and the basin's hydrological response.

The methodology used in Random Forest consisted of selecting multiple predictor variables, such as rainfall intensity and duration, land cover, temperature, and soil moisture. The model generated multiple decision trees from these variables, which were combined to obtain a robust estimation of the flows. The model was calibrated using the cross-validation technique, ensuring its accuracy and reducing the margin of error in predicting extreme hydrological events.

In addition, the results were compared with traditional deterministic models, such as HEC-RAS, evaluating the predictive capacity of Random Forest using performance metrics such as the root mean square error (RMSE) and the coefficient of determination (R^2). This allowed for quantifying the improvement in flow estimation obtained with the machine learning model.

3. Results.

3.1. Geomorphological characterization of the basin.

The morphometric analysis allowed for characterizing the hydrological dynamics of the Río Frío basin, determining its dimensions and relevant geometric relationships. A surface area of 378,699 km^2 and a perimeter of 126,114 km were obtained, indicating a moderate-sized basin with an influence on runoff and flow behavior.

Table 1 summarizes the basin's geomorphological calculations, including compaction coefficients, drainage density, slopes, and the estimated concentration time according to the Kirpich equation.

Table 1: Geomorphological calculations of the Rio Frio basin.

ABSTRACT: GEOMORPHOLOGICAL PARAMETERS OF THE RIO FRIO MICRO-BASIN

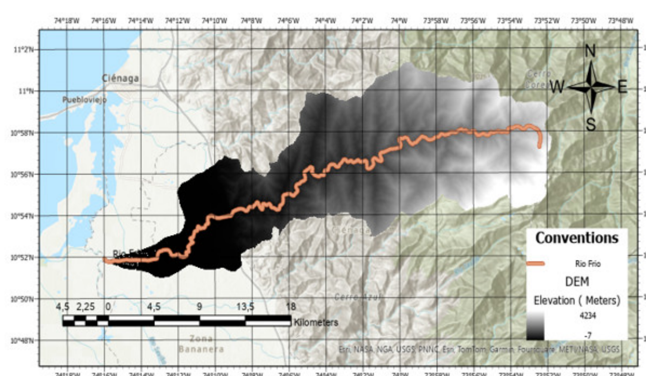
Table N° 05: Summary of the Geomorphological Calculations of the Río Frio Basin

PARAMETERS				UND	NOMENCLATURE	BASIN
Total area of the basin				Km²	At	378,699
Perimeter				Km.	P	126,114
FORM RELATIONSHIPS	BASIN FACTOR	Compactness Coefficient (Gravelius)		s/U	$Kc = 0.28\ P / (At)^{1/2}$	1,828
		FORM FACTOR	Basin Length	Km.	LB	56,871
			Average Width of the Basin	Km.	AM = At / LB	6,659
			Form Factor	s/U	Kf = AM / LB	0,117
	Equivalent rectangle		Older Side	Km.	$Kc^*(pi^*A)^{1/2}/2^*(1+(1-4/pi^*Kc^2))$	56,871
			Minor Side	Km.	$Kc^*(pi^*A)^{1/2}/2^*(1-(1-4/pi^*Kc^2))$	6,659
	Drainage density			Km./Km².	Dd = Lt / At	1,533
Drainage density Total slope of the basin				Km.	Ht	4,241
Average height of the basin				m.s.n.m.	Hm	1433
Slope basin (Met. Rectangle Equivalent)				%	Ht / Lma	7,46%
Concentration Time According to Kirpich				min.	0.0195(L^3/h)^0.385	242,86

Source: Authors.

Figure 2 shows the **basin's altimetric distribution**, allowing us to visualize the variability of the relief and its influence on hydrological behavior.

Figure 2: The digital elevation map of the Rio Frio basin.

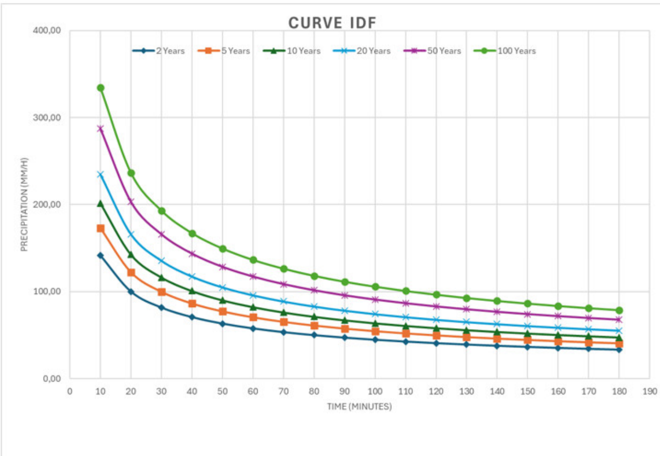


Source: Authors.

3.2. IDF Curves

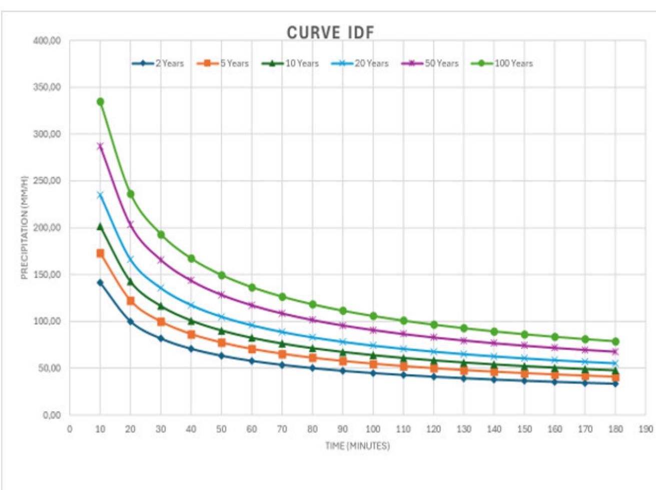
Intensity-duration-frequency (IDF) curves were generated for the basin using historical precipitation data from IDEAM's El Enano (code 29060160) and San Isidro (code 29060280) meteorological stations. These curves allow the evaluation of the magnitude of extreme precipitation events and their impact on the basin's hydrological response.

Figure 3: IDF curve at El Enano station. 7904 average annual maximum precipitation data were taken from 2000 to 2019.



Source: Authors.

Figure 4: IDF curve at San Isidro station. 7,422 data points were taken for average annual maximum precipitation from 2000 to 2019.



Source: Authors.

3.3. Population projection and water demand.

The population projection for the municipality of Zona Bananera shows sustained growth over the next 25 years, with an annual growth rate of 1.02%. Using the geometric method, it was estimated that the population would reach 99,188 inhabitants in 2048, generating a significant increase in the demand for drinking water. Table Table 2presents the population projection for the area, showing a growth of 44.3%.

3.4. Hydrodynamic modeling of the Frío River.

Hydrodynamic modeling of the Frío River was carried out using HEC-RAS 6.4.1 software, which allowed the evaluation of flow variability and flow velocities in different river sections for various return periods. This modeling identified areas with greater susceptibility to erosion and overflow, providing key information for the planning and management of water resources in the basin.

Table 2: Population projection of the Banana Zone, Magdalena.

Growth rate	**1.02 %**	
Initial population	**68722 (Hab)**	
Time (Years)	**Year**	**Future population (Inhab)**
5	2028	80968
10	2033	85182
20	2043	94281
25	2048	99188

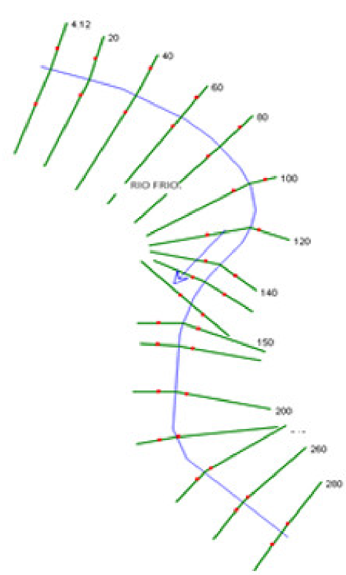
Source: Authors.

3.4.1. Channel geometry and hydraulic parameters.

For the model configuration, Manning's roughness coefficients were established based on the table proposed by Chow (2004). A value of 0.035 was assigned for the river bed, characterized by rocky bottoms and banks with weeds, and a value of 0.05 for the flood plain, corresponding to areas with dense vegetation.

In addition, the energy loss coefficients between adjacent sections were defined, with values of 0.1 for gradual contractions and 0.3 for expansions, ensuring the accuracy of the hydraulic analysis of the study section. Figure Figure 5 the geometry of the Frío River modeled in HEC-RAS, where the evaluated cross sections are represented.

Figure 5: The geometry of RIO FRIO, modeled in HEC-RAS 6.4.1.

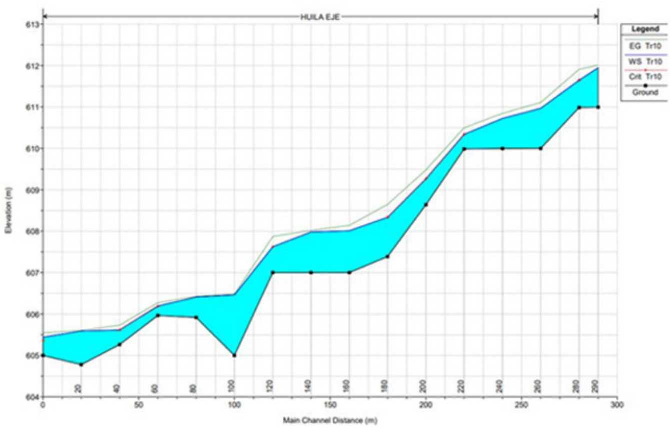


Source: Authors.

3.4.2. Longitudinal flow profile.

The model allowed for a detailed representation of the flow under maximum conditions, facilitating the identification of water level changes along the study section. Figure Figure 6the longitudinal profile of the flow in the Río Frío stream under maximum flow conditions.

Figure 6: Longitudinal profile of the flow in the Río Frío stream under maximum conditions.



Source: Authors.

Based on the flow analysis in the most critical cross sections, the variation in hydraulic levels at the points of most significant risk of overflow was identified. These results are essential for designing mitigation strategies for extreme events.

3.4.3. Hydraulic conditions and estimated flow rates.

The results of the HEC-RAS modeling allowed for obtaining values of total flow, flow velocity, and the river's surface width under different return periods. These data are essential to evaluating the channel's hydraulic capacity and response to high precipitation events.

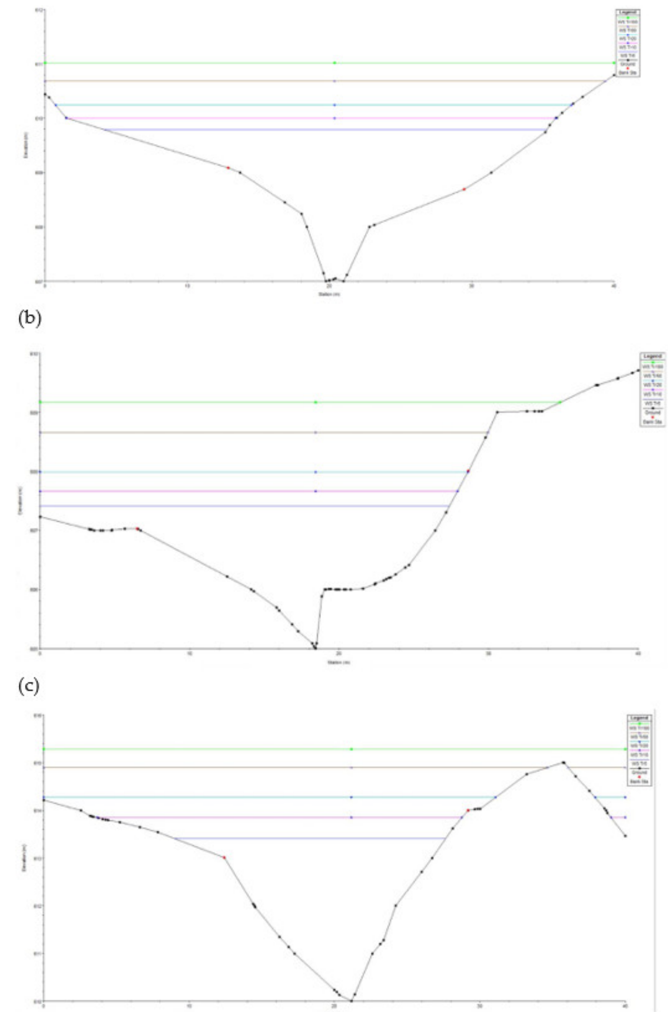
Table 3: Results of HEC-RAS modeling in the Banana Zone, Magdalena.

Return Period (years)	Total flow rate (m^3/s)	Flow Velocity (m/s)	Surface Width (m)
5	194.17	4.72	26.56
10	248.08	5.35	33.3
20	325.28	5.83	34.76
50	494.08	6.65	34.76
100	645.10	7.25	34.76

Source: Authors.

Additionally, the HEC-RAS 6.4.1 model shows the flow passage through each section, allowing the maximum water level in the study sector to be observed. Figure 7 shows the cross sections with their respective levels.

Figure 7: Cross-section of the flow in the critical sections of the Río Frío stream. (a) Cross-section of the crucial point at elevation 180, critical regime; (b) subcritical regime at elevation 160; (c) supercritical re-gime at elevation 265.



Source: Authors.

The results reflect the relationship between the increase in flow rate and the speed of the flow as a function of the return period, which directly influences the risk of erosion and overflow in the basin's most vulnerable areas.

3.5. Comparison between hydrodynamic models.

To assess the accuracy of flow estimation in extreme events, we compared the results obtained with the deterministic HEC-RAS model and the Random Forest machine learning model. This comparison allowed us to analyze the advantages and limitations of each method in managing water resources in the Río Frío basin.

The HEC-RAS 6.4.1 model solves equations of gradually varying flow, allowing the simulation of the river's hydraulic response in different hydrometeorological scenarios. In contrast, Random Forest employs an approach based on the analysis of patterns from historical precipitation data and hydrometeorological records from IDEAM, using variables such as maximum daily precipitation, duration of rainfall events, land cover, and terrain slope.

The results show that Random Forest improves the flow prediction accuracy, providing estimates closer to the actual values. Table Table 4 the flow values estimated by each model for different return periods.

Table 4: Estimated flows with HEC-RAS and Random Forest.

Return Period (years)	Estimated HEC-RAS Flow Rate (m^3/s)	Estimated Flow Random Forest (m^3/s)
5	194.17	189.12
10	248.08	242.87
20	325.28	318.45
50	494.08	480.67
100	645.10	630.21

Source: Authors.

The values obtained show that Random Forest provides slightly lower estimates than HEC-RAS, reducing the overestimation of flow that can occur in deterministic models.

Error (RMSE) and coefficient of determination (R^2) Table 5.

Table 5: Performance comparison between HEC-RAS and Random Forest.

Model	RMSE (m^3/s)	R^2	Precision in extreme events
HEC-RAS	0.7	0.89	Moderate
Random Forest	0.595	0.98	High

Source: Authors.

The results show that Random Forest obtained a coefficient of determination of 0.98, indicating a better predictive capacity than HEC-RAS (R^2 of 0.89). This result suggests that machine learning can improve the prediction of extreme flows, optimizing water resource management and hydraulic infrastructure planning.

4. Discussion.

The hydraulic analysis of the Frío River shows a progressive increase in the elevation of the water surface with the increasing flow in all sections, evidencing the channel's capacity to transport flow in extreme events. In axis 290, the water level rises from 615.03 m in Tr5 to 617.00 m in Tr100. The channel's longitudinal profile presents slope variations, with a supercritical flow in the upper part of the basin and a lower inclination in the lower part. The flow velocity increases with the flow, reaching values of 4.92 m/s in Tr5 and 7.25 m/s in Tr100 on axis 290, suggesting a more significant potential for erosion in extreme events.

The energy gradient is low, although, in specific sections such as axes 100 and 260, where the slopes vary between 0.04 and 0.09, more incredible energy is observed available for the flow, which can generate rapid flows and localized erosion. The relationship between velocity and flow area indicates that narrower sections, such as axis 260, are more efficient in hydraulic conduction, while others, such as axis 290, can promote sedimentation zones. In axis 100, the Froude number decreases from 2.23 in Tr5 to lower values in Tr100, evidencing a transition from supercritical to subcritical flow due to geometric variations. In axes 0 and 20, the increase in critical levels between return periods is minimal (0.10 m). In contrast, on axis 240, more significant increases are observed (0.30 m), suggesting a lower sensitivity to extreme flows in the upstream areas.

Sections 140 and 240 show minimal differences between available energy and water level, indicating more critical conditions and a hydraulic capacity at its limit. In axes 290 and 280, hydraulic widths remain relatively constant, suggesting a stable channel geometry due to geological conditions or previous interventions. Sections with larger hydraulic widths, such as axes 120 and 60, can store water laterally in extreme events, helping flood mitigation. In axis 100, the water level in Tr100 (613.09 m) approaches the containment limit (610.42 m), indicating a high risk of overflow due to its steep slope. In axes 240 and 260, velocities exceed 8 m/s in Tr100, representing a significant risk of scour for fluvial structures and retaining walls.

4.1. *Random Forest Modeling Results.*

The Random Forest machine learning model was applied and trained with historical precipitation data and hydrometeorological records from IDEAM to improve the prediction of flows in extreme events. The variables used included maximum daily precipitation, rain event duration, land cover, and terrain slope. The results obtained with Random Forest show greater accuracy in predicting flows compared to HEC-RAS. It is observed that the Random Forest model tends to provide slightly lower estimates compared to HEC-RAS, reducing the overestimation of flows characteristic of deterministic models.

4.2. *Comparison between HEC-RAS and Random Forest in Flow Prediction.*

To evaluate the performance of both models, key metrics such as root mean square error (RMSE) and coefficient of determination (R^2) were calculated. A 15% reduction in RMSE was observed concerning HEC-RAS calculations, indicating greater accuracy in predicting extreme flows. In addition, the coefficient of determination (R^2) increased by 10%, reflecting a greater capacity of the model to explain data variability. More excellent stability in prediction was also evident, especially in intense precipitation events.

Random Forest represents a step forward in the hydrological modeling of the Frío River, providing an efficient and accurate alternative for estimating flows and improving water resource management in the region. Its integration with traditional approaches such as HEC-RAS allows for more robust analysis, combining the rigor of physical models with the flexibility of artificial intelligence for decision-making in water planning and management.

Conclusions.

This study demonstrates the importance of integrating traditional hydraulic models such as HEC-RAS with advanced machine learning techniques such as Random Forest to improve flow prediction in extreme events. Hydraulic modeling reveals the vulnerability of specific stretches of the Frío River to erosion and overflow processes, highlighting the need for mitigation strategies supported by accurate data and efficient models.

Implementing Random Forests allows significantly reduced error in estimating flows, achieving more stable and reliable predictions compared to HEC-RAS. Integrating this approach with historical hydrometeorological data is a key tool for anticipating hydrological impacts and optimizing decision-making in water resource management, especially in regions prone to extreme climate variations.

For this reason, the combination of deterministic models and artificial intelligence represents an innovative approach to hydrological modeling, providing a robust framework for water planning and management in river basins. This type of study contributes to formulating adaptation and mitigation policies against climate change, promoting the sustainability of water resources, and strengthening the resilience of communities dependent on the Frío River.

Despite the progress made in the hydrological and hydraulic modeling of the Frío River, the study has some limitations that should be considered for future research. The availability and quality of hydrometeorological data is one of the main restrictions since historical information on precipitation and flows does not uniformly cover the entire basin. The lack of monitoring stations in key areas makes validating the models used more accurately tricky.

Another limitation is the temporal and spatial scale of the models used. While Random Forest improved the accuracy of streamflow predictions, its performance is highly dependent on the representativeness of the training data. Furthermore, the comparison with HEC-RAS shows that, although traditional models offer a solid basis for hydraulic simulation, they may not capture the non-linear variations of the water system as effectively in extreme events.

For future research, integrating hybrid models that combine artificial intelligence with deterministic techniques is recommended, as well as implementing additional monitoring stations to improve the calibration and validation of the models. It is suggested to expand the coverage of the study by incorporating other climatic and land-use variables that may influence the dynamics of the basin.

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