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A New Funnel Sliding Mode Control for Autonomous Underwater Vehicles in the Vertical Plane

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ABSTRACT

In this paper, a new procedure for depth control of Autonomous Underwater Vehicles is proposed for the vertical tracking. This procedure is based on the combination of sliding mode control and funnel control. A variable funnel is used to transform tracking error fall within the funnel boundary. Numerical simulations show the effectiveness of proposed control framework, it is demonstrated that proposed controller has accurate good tracking performance, which can help inform decisions regarding control techniques for regulating the vertical position of underwater vehicles.

1. Introduction.

Control for Autonomous Underwater Vehicles (AUVs) is a problem of interest today due to its multiple applications in underwater tasks, such as deep-sea inspections, pipeline inspection, oceanographic mapping, and some military applications, including detecting, locating, and neutralizing undersea mines. Designing a robust controller for Autonomous Underwater Vehicles navigation is a challenging task; model dynamics of the AUV contain high-order nonlinearities due to coupling effects and there exist uncertainties due to the limited knowledge of the hydrodynamics and buoyancy forces affecting the system.

Many research works have been conducted to solve and control AUVs and marine vehicles with different techniques to overcome the aforementioned challenges; an Integral Sliding Mode (ISM) controller is presented in [1] to enhance time delay controller in order to improve control precision. PID techniques are used to develop the design of separate controllers [2], a decoupled PD controller is considered to solve the orientation problem and position for AUVs [3], sliding mode tracking control is presented in [4, 5, 6, 7], numerical simulation results demonstrate that the proposed controller achieves precise tracking for underactuated AUVs. Second-order sliding mode controller is proposed in [8] to stabilize an AUV, whereas adaptive tracking control is presented in [9, 10, 11, 12, 13], learning control for Underwater Robotic Vehicles (URVs) is studied in [14], Neural network control [15, 16, 17, 18], fuzzy control [19, 20], Lyapunov-based techniques [21, 22] and Lyapunov's direct method [23]. In [5], a trajectory tracking sliding mode controller is designed for autonomous surface vessels with the use of nonlinear hydrodynamic damping model. Nonlinear underactuated tracking control techniques are used in [21, 22] to develop trajectory tracking controllers for underactuated ships. Lyapunov's direct method is used in [23] to solve the trajectory tracking control problem of underactuated ships.

Recently, a new algorithm is considered for improving and shaping the transient response for a class of nonlinear systems. The idea behind this algorithm is to design an appropriate funnel [24]. Funnel-based control strategy, is a high gain-based time-varying control strategy which guarantees 'tracking with prescribed transient accuracy'. It has been proven that funnel control is an appropriate tool for many practical systems like

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chemical reactor models [25], speed control of wind turbines [26], voltage and current control of electrical circuits [27], and control of Peak Inspiratory Pressure (PIP) [28], the time delay estimation (TDE) and the back-stepping approach are used by Gun Rae Cho et al. [30] for the tracking of torpedo-shape Autonomous Underwater Vehicles (AUVs), the same combined method is proposed for the control of an Autonomous Underwater Vehicle (AUV) [31]. A combined funnel-based control and a sliding mode control method is used by [32] for the control of an AUV in the vertical plan. Fatima Zohra Kadri et al. [33] used.

A Model Reference Adaptive Control (MRAC) approach based on multilayer per-ceptron (MLP) neural networks to control the depth of a REMUS Autonomous Underwater Vehicle.

The purpose of this paper is to develop a new proposal for a combination between funnel-based control and sliding mode control of an AUV in the diving plane, where the output variables are the forward velocity and the vehicle's depth. It is considered that the depth subsystem, which has relative nature degree too. Based on these dynamics, a funnel-based sliding mode control is designed. The funnel-based control has been designed as a time-varying coefficient of the sliding surface. This idea leads to an additional control term, and it also improves the performance of the proposed controller. Moreover, a control design for the vehicle forward velocity is also considered, while in most previous works, they have considered such variable as constant.

The remainder of this contribution in this paper is categorized as follows. We begin by configuration and modelling of the submarine motion equations. In Section 2, a model of the AUV in the horizontal plane is presented. The control design is then proposed in Section 3. In Section 4, the performance of the proposed control scheme is validated using computer simulations. Finally, the conclusions of this work are summarized in Section 5.

2. AUV Modelling.

In [29], the submarine motion equations have been developed and introduced. The complete dynamics of motion for AUV as a six-degrees of freedom (DOF) rigid body moving in an ideal fluid are described by a set of 12 nonlinear, coupled, first-order differential equations with constant coefficients. To simplify the control law design procedure, we restrict our attention to the dive plane with horizontal plane-control surfaces at zero. Then, the heave and pitch equations of motion of the vehicle in the body-fixed coordinate frame are given as follows:

$$\begin{aligned} m(\dot{u}+qw) = {} & X_{qq}q^2 + X_{\dot{u}}\dot{u} + X_{wq}wq + X_{q\delta}uq\delta \\ + {} & X_{ww}w^2 + X_{w\delta}\delta uw + X_{\delta\delta}u^2\delta^2 \\ - {} & (W-B)\sin\theta + F_P \end{aligned} \quad (1)$$

$$\begin{aligned} m(\dot{w}+uq) = {} & Z_{\dot{q}}\dot{q} + Z_{\dot{w}}\dot{w} + Z_{uq}uq + Z_{uw}uw \\ + {} & Z_{uu}u^2\delta + Z_{w|w|}w|w| + Z_{q|q|}q|q| \\ + {} & (W-B)\cos\theta + Z_H \end{aligned} \quad (2)$$

$$\begin{aligned} I_{yy}\dot{q} = {} & M_{\dot{q}}\dot{q} + M_{\dot{w}}\dot{w} + M_{uq}uq + M_{uw}uw + M_{uu}u^2\delta \\ - {} & (z_G W - z_B B)\sin\theta - (x_G W - x_B B)\cos\theta \\ + {} & M_{w|w|}w|w| + M_{q|q|}q|q| + M_P \end{aligned} \quad (3)$$

$$\dot{\theta} = q \quad (4)$$

$$\dot{z} = -u\sin\theta + w\cos\theta \quad (5)$$

Where u is the vehicle's forward velocity, ω is the heave velocity, θ is the pitch angle, q is the pitch angle velocity, z is the vehicle's depth, δ is the control fin angle, F_P is the propulsion force that control the forward velocity, m is the mass of the vehicle, I_{yy} is the moment of inertia of the vehicle about the pitch axis, W denotes the vehicle's weight and B is the vehicle buoyancy. $Z_{\dot{q}}$, Z_{uq}, $Z_{\dot{\omega}}$, $M_{\dot{q}}$, etc., are the hydrodynamics parameters. Finally, M_P and Z_H represent the cross-flow drag terms and are considered as disturbances.

Defining the state vector $x = [x_1,\ x_2,\ x_3,\ x_4,\ x_5]^T = [u, \omega, q, \theta, z]^T$, the control vector $v = [F_P,\ \delta]^T$ and the inertia matrix is:

$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 & 0 & 0 & 0 \\ 0 & m - X_{\dot{w}} & -Z_{\dot{q}} & 0 & 0 & 0 \\ 0 & -M_{\dot{w}} & I_{yy} - M_{\dot{q}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

The vehicle dynamics (1) are represented in the state space formats

Or

$$M\dot{x} = f_x(x,t) + g_x(x,v) \quad (7)$$

$$\dot{x} = f(x,t) + g(x,v) \quad (8)$$

With outputs:

$$y_1 = u$$

$$y_2 = z$$

Where: $f_x(x,t) = M^{-1}\ f_x(x,t)$ and $g_x(x,v) = M^{-1}\ g_x(x,v)$

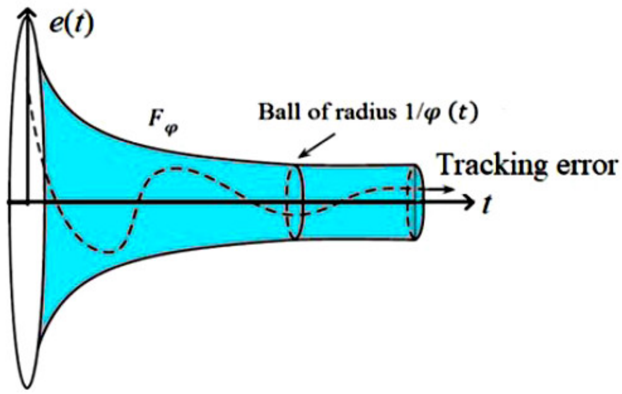
2.1. Control Design.

In this section, some necessary definitions and theorems are given.

Definition 1: The performance funnel F_φ (see Figure 1) is determined by a bounded function $\psi(t)$ where $\psi(t) = \frac{1}{\varphi(t)}$ and:

$$F_\varphi := \{(t,e)\,\epsilon R\, \|e(t)\| < \psi(t)\}$$

Figure 1: The funnel F_φ and the tracking error $e(t)$ inside it.



Source: Authors.

The idea behind the funnel-based control is to shape the transient response of the tracking error to place it in the funnel. Figure 1 shows the structure of the funnel F_φ and the path of the output tracking error ($e(t)$) within it. In this approach, the shape of the funnel (i.e., F_φ) is effective on the characteristics of transient response like overshoot, rise time, settling time, and so forth.

L and $\varphi(t)$ is the function of the following class:

$$\Phi := \left\{\varphi(t)\, \epsilon w^{1,\infty}(R_+, R_+)\, |\forall t > 0 : \varphi(t) > 0\right\}$$

And $\lim_{\tau\to x} inf\varphi(\tau) > 0, \forall\tau > 0 : \varphi^{-1}(.)$ is globally Lipschitz.

where $w^{1,\infty}(R_+, R_+)$ represents a class of functions with bounded derivatives. The boundaries of the funnel (which is $\psi(t)$ in Figure 1) are affected by the appropriate choice of the function $\varphi(t)$ and the aim is that the error stays inside the funnel F_φ: Suppose

$$\lambda_1 = sup_{t\in|0,x)}\psi(t), \phi^* := inf_{t\in|0,x)}\psi(t)$$

Then $\varphi(t) \geq \frac{1}{\lambda_1}$; $\forall t \geq T$ where T is sufficiently large, then $e(t)$ is ultimately bounded by λ_1. In [24], a funnel-based controller with the structure $u(t) = -k(t)e(t)$ was employed to solve the problem of output tracking for a dynamical system with relative degree one where $k(t)$ is a time-varying gain that is dependent on the funnel shape and is selected as:

$$k(t) = \frac{\varphi(t)}{1-\varphi(t)|e(t)|} = \frac{1}{\psi(t) - |e(t)|} \tag{9}$$

By using this controller, when the error approaches the boundary of the funnel, the gain increases and prevents the error from contacting the funnel boundary, and in comparison with the adaptive approach, it is not dynamically generated.

2.2. Velocity Control.

As outlined above, most of the previous research considered the forward speed to be constant. Though, to attain that, control F_P should be correctly designed. Consider the output variable $\overline{x_1} = y_1 = u$ and the constant reference u_d. Since x_1 has relative degree one, a sliding manifold is designed as

$$\sigma_1 = \overline{x_1} - u_d \tag{10}$$

Using time derivative, equation (10) become:

$$\dot{\sigma_1} = f_u(x, t, \delta) + g_u F_p \tag{11}$$

The control law is proposed as a combination of funnel based-control and sliding mode algorithms as

$$F_P = v_{eq} + v_1 + v_2 \tag{12}$$

Where:

$$v_{eq} = \frac{-k_0 z_1 - \varepsilon k_1 f_u(x, t, \delta) + \varepsilon k_1 \dot{u}_d}{\varepsilon k_1 g_u} \tag{13}$$

$$v_1 = -\beta sign(\sigma) \tag{14}$$

$$v_2 = -\frac{1}{k_1 g_u} k(t)\sigma(t) \tag{15}$$

With k_0, k_1 and β are positive constants. ε is a small positive constant.

Also, $k(t)$ is the time-varying funnel gain which is dependent on the funnel shape and the sliding surface as below:

$$k(t) = \frac{1}{\psi(t) - |\sigma(t)|}$$

Resulting in the following closed-loop system:

$$\dot{\sigma} = \frac{-k_0 z_1 + \varepsilon k_1 \dot{u}_d}{\varepsilon k_1} - \frac{1}{k_1} k(t)\sigma(t) - g_u\beta sign(\sigma) \tag{16}$$

2.2.1. Stability proof:

The proposed control law (F_p) (Equation 12) is designed based on the sliding surface is introduced in

Choose the following Lyapunov function as:

$$V_1 = \frac{1}{2}\sigma_1^2 \tag{17}$$

Differentiating with the respect to time:

$$\dot{V}_1 = \sigma_1\dot{\sigma_1}$$

$$\dot{V}_1 = \sigma_1\left[\frac{-k_0 + \varepsilon k_1 \dot{u}_d}{\varepsilon k_1} - \frac{1}{K_1}k(t)\sigma_1(t) - g_u\beta sign(\sigma_1)\right] \tag{18}$$

$$\dot{V}_1 = -\frac{k_0}{\varepsilon k_1}\sigma_1^2 + \sigma_1\dot{u}_d - \frac{1}{K_1}\sigma_1^2 k(t) - g_u\beta\sigma_1 sign(\sigma_1) \tag{19}$$

$$\dot{V}_1 \leq -\left[\frac{k_0}{\varepsilon k_1} + \frac{1}{K_1}k(t)\right]\sigma_1^2 - g_u\beta|\sigma_1| \leq 0 \tag{20}$$

$$\dot{V}_1 \leq -D_1(t)\sigma_1^2 - g_u\beta|\sigma_1| \leq 0 \tag{21}$$

Inequality (21) implies that σ is bounded.

Furthermore, the bounded of u^d and $\dot{u}_d$implies the boundedness of σ.

Therefore, the stability of the velocity (1) with control (12) with control has been proved.

2.3. Depth Control.

The goal of this section is to achieve a constant reference z_d for the variable z, considering the second output variable $\overline{x}_2=y_2=z$, and taking its time derivative $\overline{x}_3=\dot{\overline{x}}_2$ as a new variable, equations of depth subsystems become:

$$\dot{\overline{x_2}}= -u\sin\theta + w\cos\theta = \overline{x_3}$$

$$\dot{\overline{x_3}}= -uq\cos\theta - wq\sin\theta + \cos\theta \ (f_w(x,t) - g_w(x)\delta) \quad (22)$$

Where the term $-\dot{u}sin\theta$ is omitted because equivalent control v_{eq}.

Defining the new variables $\overline{x}_4=q-10w$ and $\overline{x}_5=\theta$, the complete diffeomorphism results in

$$\overline{x}=\Phi(x)=\begin{bmatrix} u \\ z \\ -u\sin\theta + w\cos\theta \\ q-10w \\ \theta \end{bmatrix} \quad (23)$$

and the inverse transformation is of the form

$$x=\Phi^{-1}(\overline{x})=\begin{bmatrix} \overline{x}_1 \\ \frac{\overline{x}_3+\overline{x}_1\sin\overline{x}_5}{\cos\overline{x}_5} \\ \overline{x}_4+10\left(\frac{\overline{x}_3+\overline{x}_1\sin\overline{x}_5}{\cos\overline{x}_5}\right) \\ \overline{x}_5 \\ \overline{x}_2 \end{bmatrix} \quad (24)$$

The complete subsystem for the depth control becomes

$$\dot{\overline{x}}_1=\overline{x}_3 \quad (25)$$

$$\dot{\overline{x}}_3=\overline{f}_3(\overline{x},t)-\overline{g}_3(\overline{x})\delta \quad (26)$$

$$\dot{\overline{x}}_4=\overline{f}_4(\overline{x}_1,\overline{x}_4,\overline{x}_5,t)+\overline{g}_4(\overline{x})\overline{x}_3 \quad (27)$$

$$\dot{\overline{x}}_5=\overline{f}_5(\overline{x}_1,\overline{x}_4,\overline{x}_5)+\overline{g}_5(\overline{x})\overline{x}_3 \quad (28)$$

Where

$$\begin{aligned} \vec{f_3}(x,t) &= -uq\cos\theta - \omega q\sin\theta + \cos\theta f_\omega(x,t)dt \\ \dot{\vec{g}}_3(x) &= \cos\theta\, g_\omega(x) \end{aligned} \quad (29)$$

the second sliding manifold σ_2 is formulated as

$$\sigma_2=c_0\int(\overline{x}_2-z_d)\,dt+c_1(\overline{x}_2-z_d) \quad (30)$$

and its time derivative calculated, becomes:

$$\dot{\sigma}_2=c_0(\overline{x}_2-z_d)+c_1(\overline{x}_3-\dot{z}_d) \quad (31)$$

the control law for δ is proposed of the form

$$\delta=\delta_{eq}+\delta_1+\delta_2 \quad (32)$$

where,

$$\delta_{eq}=\frac{1}{\varepsilon^2\alpha_2\bar{g}_3(x)}\Big(-\alpha_0(\bar{x}_2-z_d)-\alpha_1(\bar{x}_3-\dot{z}_d) -\varepsilon^2\alpha_2\bar{f}_3(x,t)+\varepsilon^2\alpha_2\ddot{z}_d\Big) \quad (33)$$

$$\delta_1= -\gamma\, sign(\sigma_2) \quad (34)$$

$$\delta_2= -\overline{k}(t)\,\sigma_2(t) \quad (35)$$

With

$$\overline{k}(t)=\frac{1}{\overline{\psi}(t)-|\sigma_2(t)|} \quad (36)$$

and c_0, c_1, α_0, α_1, α_2 , γ and ϵ are design constants parameters,

So, the proposed control (32) achieves the stability of the system in the depth reference with ultimately bounded convergence.

2.3.1. Stability convergence proof:

In this section, the boundedness of all signals and the stability of the subsystem (25-26) in both the reaching phase and the sliding phase will be provided.

Choose the following Lyapunov function as:

$$V=\frac{1}{2}\sigma_2^2 \quad (37)$$

Differentiating (37) with respect to time t we have:

$$\dot{V}=\sigma_2\dot{\sigma_2} \quad (38)$$

$$\dot{V}=\sigma_2\left[c_0(\overline{x_3}-\dot{z_d})+c_1\{(f_3(\overline{x},t)-\overline{g_3}(\overline{x})\sigma)-\ddot{z_d}\}\right] \quad (39)$$

$$\begin{aligned}\dot{V} = \sigma_2\Big[& c_0(x_3-\dot{z}_d)+c_1\bar{f}_3(\bar{x},t)-c_1\left(\frac{\alpha_0}{\varepsilon^2\alpha_2}(\bar{x}_2-z_d)\right) \\ & -c_1\left(\frac{\alpha_1}{\varepsilon^2\alpha_2}(x_3-\dot{z}_d)\right)+\bar{f}_3(\bar{x},t)-\ddot{z}_d \\ & -c_1\nu\sigma(\sigma_2)-c_1\bar{k}(t)\sigma_2\Big]\end{aligned} \quad (40)$$

$$\begin{aligned}\dot{V} = \sigma_2\Big[& c_0(x_3-\dot{z}_d)+c_1 f_3(\bar{x},t)-c_1\frac{\alpha_0}{\varepsilon^2\alpha_2}(\bar{x}_2-z_d) \\ & -\frac{\alpha_1}{\varepsilon^2\alpha_2}(\bar{x}_3-\dot{z}_d)-c_1\bar{f}_3(\bar{x},t)+c_1\ddot{z}_d \\ & -c_1\nu\sigma(\sigma_2)-c_1\bar{k}(t)\sigma_2\Big]\end{aligned} \quad (41)$$

$$\begin{aligned}\dot{V} = \sigma_2\Big[& -c_1\frac{\alpha_0}{\varepsilon^2\alpha_2}(\bar{x}_2-z_d) \\ & -\left(-c_0+\frac{\alpha_1}{\varepsilon^2\alpha_2}\right)(\bar{x}_3-\dot{z}_d)+c_1\ddot{z}_d \\ & -c_1\nu\sigma(\sigma_2)-c_1\bar{k}(t)\sigma_2\Big]\end{aligned} \quad (42)$$

$$\dot{V}=\sigma_2\Big[\ -\Big(c_1\frac{\alpha_0}{\varepsilon^2\alpha_2}(x_2-z_d)+\Big(\frac{\alpha_1}{\varepsilon^2\alpha_2}-c_0\Big)(\bar{x}_3-\dot{z}_d)\Big)+c_1\ddot{z}_d-c_1\nu\sigma(\sigma_2)-c_1\bar{k}(t)\sigma_2\Big] \tag{43}$$

$$c_1\frac{\alpha_0}{\varepsilon^2\alpha_2}(x_2-z_d)+\left(\frac{\alpha_1}{\varepsilon^2\alpha_2}-c_0\right)(\overline{x_3}-\dot{z_d})\leq\sigma_2 \tag{44}$$

Then:

$$\dot{V}\leq\sigma_2\left[-\sigma_2-c_1\ddot{z_d}-c_1\nu\sigma(\sigma_2)-c_1\bar{k}(t)\sigma_2\right] \tag{45}$$

$$\dot{V}\leq-\sigma_2^2-c_1\nu|\sigma_2|-c_1\bar{k}(t)\sigma_2^2\leq0 \tag{46}$$

$$\dot{V}\leq-\left(1+c_1\bar{k}(t)\right)\sigma_2^2-c_1\nu|\sigma_2| \tag{47}$$

$$\dot{V}\leq-D_2(t)\sigma_2^2-c_1\nu|\sigma_2|\leq0 \tag{48}$$

From the Lyapunov stability theory, the boundedness of σ_2 can be ensured under the boundedness of the states system and the control input.

Theorem1:

Consider system (1-5) and we take controller (12) and (32), chosen control parameters appropriately, then the following results can be obtained:

- The reaching condition of sliding mode control is satisfied, that $\sigma_1(t)=\sigma_2(t)=0$ at $t\to+$
- The closed loops system is uniformly asymptotically stable.
- By section the different parameter values $\sigma_1(t)$ and $\sigma_2(t)$ remains inside the funnel.

Proof:

$$\text{let } \sigma=[\sigma_1,\sigma_2].$$

Define

$$\varphi_1(\|\sigma\|)=\frac{1}{4}\left(\sigma_1^2+\sigma_2^2\right) \tag{49}$$

$$\varphi_2(\|\sigma_2\|)=\sigma_1^2+\sigma_2^2 \tag{50}$$

$$W(\sigma)=T\left(\sigma_1^2+\sigma_2^2\right) \tag{51}$$

$$T=(D_1+D_2) \tag{52}$$

We choose the following Lyapunov function:

$$V=V_1+V_2=\frac{1}{2}\left(\sigma_1^2+\sigma_2^2\right) \tag{53}$$

$$\dot{V}=\dot{\sigma_1}\sigma_1+\dot{\sigma_2}\sigma_2 \tag{54}$$

$$\dot{V}\leq-D_1(t)\sigma_1^2-g_u\beta|\sigma_1|-D_2(t)\sigma_2^2-c_1\nu|\sigma_2| \tag{55}$$

$$\dot{V}\leq-D_1(t)\sigma_1^2-D_2(t)\sigma_2^2-g_u\beta|\sigma_1|-c_1\nu|\sigma_2| \tag{56}$$

$$\dot{V}\leq-D_1(t)\sigma_1^2-D_2(t)\sigma_2^2 \tag{57}$$

From the Lyapunov function (53) and its derivative (57) satisfy:

$$\varphi_1(\sigma)<V(\sigma)<\varphi_2(\|\sigma\|) \tag{58}$$

$$\dot{V}<-W(\sigma)\leq0 \tag{59}$$

Since $\varphi_1$and φ_2 are two strictly increasing function, and $W(\sigma)$ is a positive, from the Lasalle-Yoshizawa. Theorem, the boundedness of σ_1 and σ_1. Can be guaranteed.

Moreover, because $\lim_{t\to+\infty}(W(\sigma))=0$ and $\lim_{t\to0}\sigma=0$

The error dynamics system consisting $(\dot{u}-\dot{u}_d)$ $(\dot{z}-\dot{z_d})$ is uniformly asymptotically stable.

From (54) and (57)

$$\dot{\sigma}\leq A\sigma,\quad \text{where}\quad A=\begin{bmatrix}-D_1(t) & 0\\ 0 & -D_2(t)\end{bmatrix} \tag{60}$$

Solving the differential equation of (60) with initial $\sigma(t_0)$ we can obtain:

$$\sigma\leq exp\left(A\left(t-t_0\right)\right)\sigma(t_0) \tag{61}$$

Since the engine values of matrix, A are all negative, the exponential convergence of σ to zero guaranteed. Therefore, from (61) it can be found that $\sigma(t)$ remains inside the funnel by the choice of the control parameters values in $D_1(t)$ and $D_2(t)$.

Furthermore, due to (equation σ_1) and (equation σ_2), when the error states are trapped in the sliding surface of $\sigma=0$, then asymptotical stability errors can be guaranteed.

3. Simulation Results.

In this section, computer simulations are presented to confirm the efficiency and proper performance of the proposed control law. the AUV model parameter as presented in Table 1. In these simulations, initial state is taken to be zero. The related parameters in the simulation task are also presented in Table 1. The forward velocity u reference is set to $2\frac{m}{s}$ and the depth z reference to 20 m.

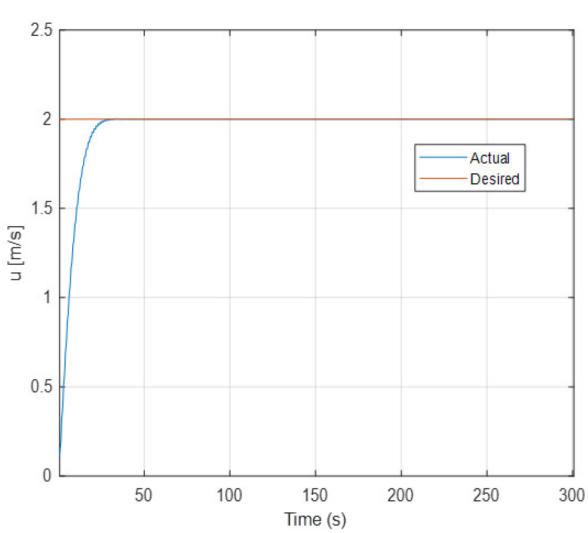
Table 1: AUV model parameters and controller parameters design.

Parameter	value	Parameter	value
W	$299\ N$	Z_{uq}	$-5.22\,\frac{kg}{rad}$
m	$30.48\ kg$	$Z_{u\omega}$	$-28.6\,\frac{kg}{m}$
I_{yy}	$3.45\ kg.m^2$	Z_{uu}	$-6.15\,\frac{kg}{m.rad}$
z_G	0	$Z_{\omega\|\omega\|}$	$-131\,\frac{kg}{m}$
z_B	$-0.0196\ m$	$Z_{q\|q\|}$	$-0.632\ kg.\frac{m}{rad^2}$
x_G	0	$M_{\dot{q}}$	$-4.88\ kg.\frac{m^2}{rad}$
y_G	0	$M_{\dot{\omega}}$	$-1.93\ kg.m$
x_B	0	M_{uq}	$-2\ kg.\frac{m}{rad}$
y_B	0	$M_{u\omega}$	$24kg$
B	$306\ N$	M_{uu}	$-6.15\,\frac{kg}{rad}$
X_{qq}	-1.5	$M_{\omega\|\omega\|}$	$-3.18\ kg$
$X_{\dot{u}}$	-16	$M_{q\|q\|}$	$-188\ kg.\frac{m^2}{rad^2}$
$X_{\omega q}$	-20	k_0	1
$X_{q\delta}$	2.5	k_1	5
$X_{\omega\omega}$	17	ε	0.01
$X_{\omega\delta}$	4.6	β	6
$X_{\delta\delta}$	1	c_0	1
$Z_{\dot{q}}$	$-1.93\ kg.\frac{m}{rad}$	c_1	0.5
$Z_{\dot{\omega}}$	$-35.5\ kg$	α_0	1.1

Source: Authors.

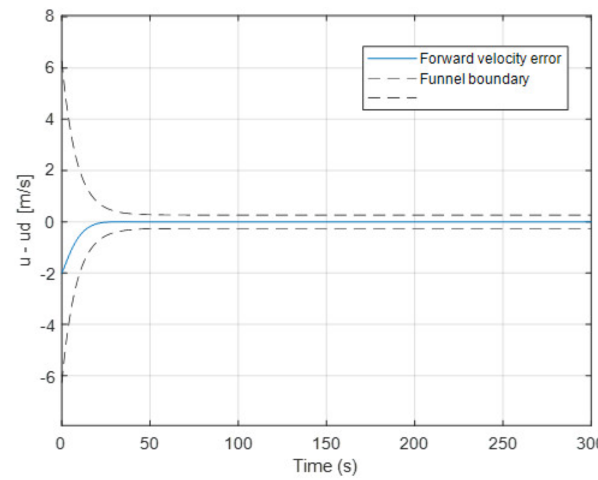
First, the results for ideal model (no disturbances and no uncertainties) are presented. Figure 2 shows the first output, namely, the forward velocity u performance and how it achieves the reference in a short time. Figure 3 shows the response of the vehicle depth, which achieves the reference with very little overshoot.

Figure 2: The actual and desired Forward velocity of the AUV.



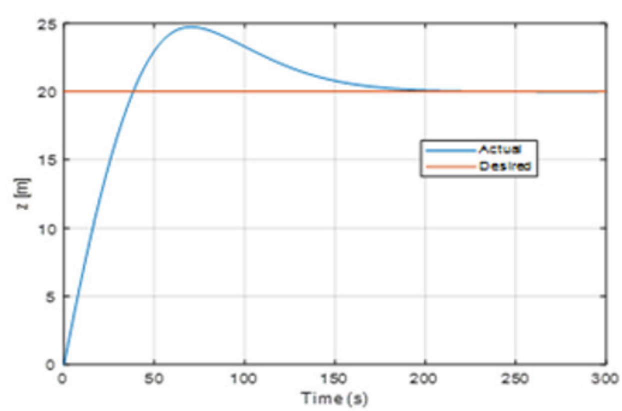
Source: Authors.

Figure 3: Time response of Forward velocity error.



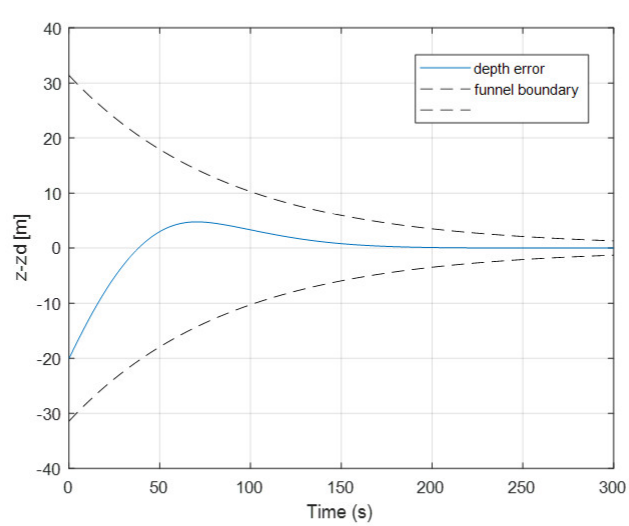
Source: Authors.

Figure 4: The actual and desired depths of the AUV.



Source: Authors.

Figure 5: Time response of depth error.

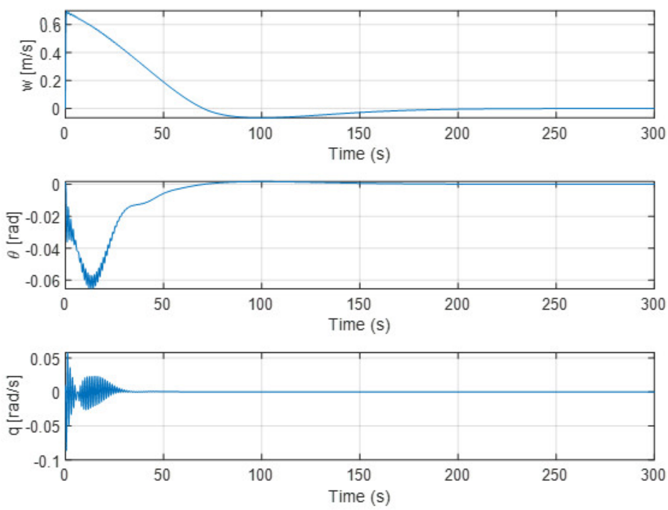


Source: Authors.

In Fig. 6, the performance of the remaining state variables is displayed, where it can be seen that these variables achieve steady state, there is no instability.

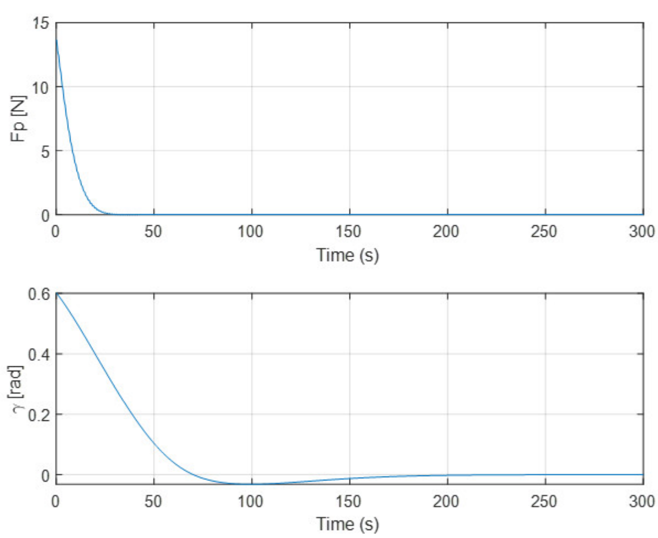
Figure 7 shows the response of the resultant control laws applied to the system.

Figure 6: States ω, θ and q responses.



Source: Authors.

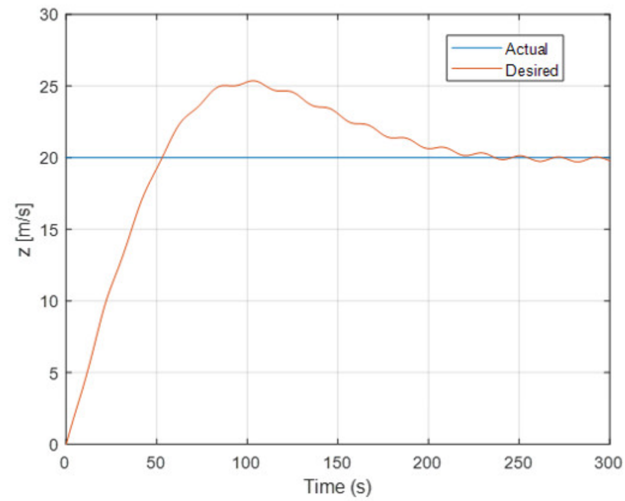
Figure 7: Control inputs.



Source: Authors.

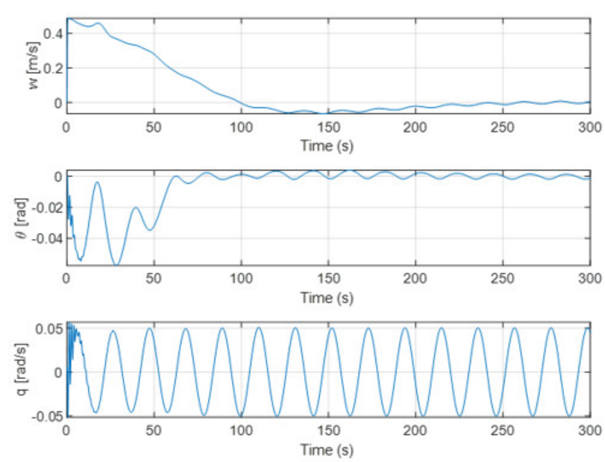
To display the effectiveness of our proposed controller scheme in presence of model's uncertainties and disturbances, in numerical simulations, we add noise on each hydro-dynamic coefficient to simulate model's uncertainties. Moreover, we use a sea current to simulate environmental disturbances.

Figure 8: The actual and desired depths of the AUV in presence of the disturbances and parameters uncertainties.



Source: Authors.

Figure 9: States ω, θ and q₋ responses in presence of disturbances and parameters uncertainties.



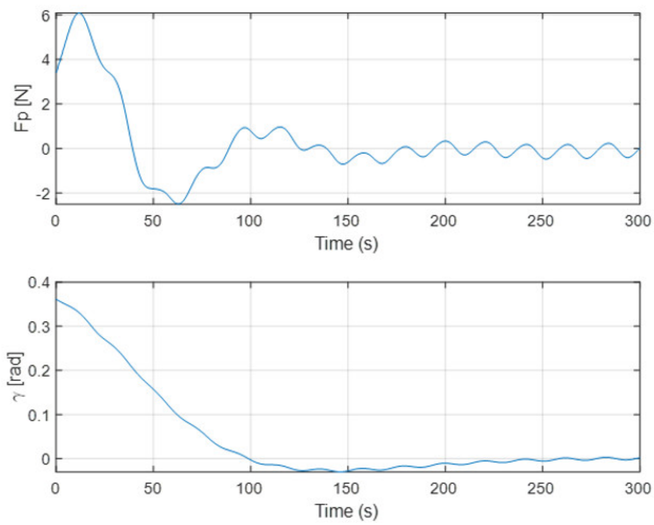
Source: Authors.

Figure 8 shows how the depth z achieves the reference with a satisfactory performance despite the disturbances and parameters uncertainties. Again, there mining state variables responses are shown in Figure 9, is successfully treated. Applied control laws responses are shown in Figure 10, where it can be seen that their amplitude remains close to the ones obtained in the absence of disturbances.

Conclusions.

In this paper, the funnel-based sliding mode controller is successfully used to control an Autonomous Underwater Vehicle in the vertical plane. In this approach, an additional term (based on the funnel) is added to the sliding mode controller

Figure 10: Control inputs in presence of disturbances and parameters uncertainties.



Source: Authors.

which leads to the improvement of the properties of the closed-loop system in different aspects. We used computer simulations which are performed by MATLAB Software. The simulation results show the effectiveness of the proposed controller in the presence of external disturbances and plant parameter variations.

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