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Effect of Middle Tension on Dynamic Behavior of Marin Risers

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ABSTRACT

Oil extraction in deep waters requires the use of specialized equipment and tools that enable safe activity in the deep. Amongst this specialized equipment is the semi-submersible platform, which is held still in the desired position through the use of cable and chain mooring. Another important component is the riser, which is a cylindrical beam used to connect the wellhead to the platform. This study includes the mathematical modeling of a typical riser and an analysis of its vibration in two planes due to platform motion, waves and current. In addition, the effects of middle tension and variation in mechanical and environmental parameters on the dynamic behavior are considered.

1. Introduction

Exploration of oil and gas fields is moving into deeper waters. In the last few years drilling activities took place in water depths of more than 2000 meters, and it is expected that this is not the limit, since fields in 3000 meters have already been discovered. In order to convey the hydrocarbon to the sea level, a steel pipe, conventionally referred to as a riser, is installed between wellhead at the seabed and floating platform. The riser is subjected to shear and oscillatory flows due to currents and waves, flows with a very high degree of complexity, with intensity and direction changing with the depth of water.

Vortex-induced vibration (VIV) is a direct consequence of lift and drag oscillations due to the vortex shedding formation behind risers. In such conditions, a better comprehension of the vortex dynamics causing vibration and fatigue of risers is essential. Also when the frequency of vortex shedding coincides with the structural natural frequency of the risers, the VIV occurs with possible high dangerous amplitudes that may lead to the failure of the risers.

An overview of recent research on the basic VIV phenomenon and empirical models relevant for slender marine structures is found in Williamson et al. (2004), Sparpkaya (2004), Gabbai et al. (2005).

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The interaction between the currents and cylinder was discussed when the fluid flows through the cylinder, and the wake oscillator model, correlation model, and statically model were presented for the prediction of dynamic response of cylinders in the non-uniform flow by Sparpkaya (1979). Also Iwan (1981) employed the wake oscillator model to predict the vibration of cylinders in the non-uniform flow.

Newman et al. (1997) and Mathelin et al. (2005) studied the effect of cylinder motion on the hydrodynamic forces. Lyons et al. (1986) expressed the fluid dynamic force in the square of velocity in Morison's equation and simplified the nonlinear term into linear term.

There have been also lots of experimental studies for model and full-scale risers, see Vikestad et al. (2000), Bearman (1984), Parkinson (1989), Khalak (1999) and Govardhan et al. (2001).

Yan-Qiu et al. (1992, 1994) simplified the riser into the simple support model, and the vibration of riser induced by vortex and stability of vibration were investigated considering nonlinear fluid damping force. Chi et al. (2000) suggested the beam model with the upper moveable support and fixed support at the bottom. Also Tang et al. (2006) developed the model and calculating method of vortex-excited vibration for casing pipe in the deep water.

Chakrabarti (1980), Patel et al. (1984), Chung et al. (1996) and Hong (1995) used two methods of finite element and finite difference to predict riser response to hydrodynamic forces.

Hsu (1975) was one of the first who analyzed parametric resonance for offshore cable applications. Patel et al. (1991)

studied dynamic behavior of buoyant platform tethers assuming a constant pretension over the height and considering combined lateral and axial excitation at the top. The partial differential equation, governing the transverse motion of tether, was reduced by them to a set of Mathieu equations with the help of the Galerkin method.

For a submerged cable of 1000 meters in length, parametrically excited at the top, Chatjigeorgiou et al. (2002) studied numerically the transverse motion of the cable based on the first four modes taking the coupling between these modes into account. Suzuki et al. (2004) performed a small-scale test in air with a Terfon tube of 11 meters long that was parametrically excited at the top. Chatjigeorgiou et al. (2005) found after extensive mathematical manipulations a closed-form solution for a parametrically excited riser based on the first two modes. Kuiper et al. (2008) considered the riser stability of the straight equilibrium in calm water and used a numerical technique based on the Floquet theory.

The aim of this study is to predict dynamic behavior of the riser in synchronic occurrence of wave, current, platform excitation and middle tension. Also the natural vibration frequencies are obtained, and the response including primary resonance and the composite resonance under combined wave-current are investigated.

In this paper, a production riser for a semi-submersible platform is modeled based on a spectral method solution of the beam equation where the current and wave forcing is allowed to vary in time and over depth. It is assumed that the physical properties of the marine riser, e.g. mass, stiffness, etc., are constant along the length of the riser. Thus, the resulting coupled partial differential equations involve two independent variables, time and location along the axis.

The paper is organized as follows: Section 2, the equations of motion is presented of a submerged riser and the main assumptions are discussed with which this equation is applicable. Section 3, the results of analysis is discussed and the effect of different parameters, specially the middle tension, on dynamic behavior of marine riser is studied. Section 4 gives the concluding remarks.

2. Assumptions and equations

The considered riser is a tensioned pipe of a finite length, as sketched in Fig.1. It is assumed that wave and current flow in same direction w and the cross-section of riser are uniform. The coordinate original point is at the sea level. The top of the riser is connected to the floating platform by means of a heave compensator, which has two functions. First, this devise provides a large static tensile force at the top of the riser to avoid buckling. Secondly, it reduces the longitudinal stress variation induced by the relative vertical motion of the platform and the riser. As commonly used, the heave compensator is modeled as a spring. The connection between riser and wellhead at the bottom can be modeled as a hinge. As the surge frequency of the platform is much less than the frequency of the riser deflections, so the effect of surge motion can be considered as a static offset, w_0.

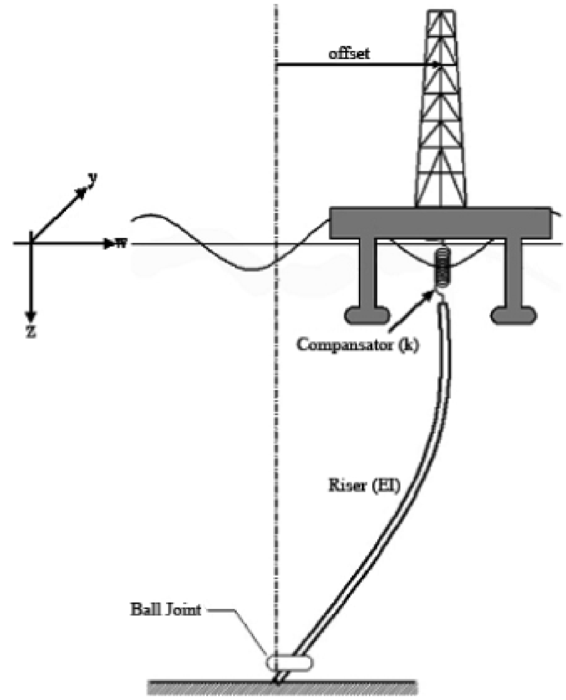

Fig. 1. Coordinate system and model.

The effective tension can be expressed as

$$T_{e(z,t)} = W_B(-z + fl) - k\Delta \quad (1)$$

where W_B is the submerged weight per unit length, f is a dimensionless pretension factor (usually 1.3), l is the length of the riser, k and Δ are the stiffness and elongation of the heave compensator (spring), respectively.

It is a common practice to define k as a function of critical amplitude of platform heaves a_c and submerged weight of the riser as

$$k = \frac{lW_B}{a_c} \quad (2)$$

It is assumed that the platform heaves harmonically with a horizontal displacement. The elongation of spring, Δ, can be expressed as

$$\Delta = \left[\left(l + a\, Cos(K w_0 - \sigma t)\right)^2 + w_0^2 \right]^{1/2} - l \quad (3)$$

where w_0 is the horizontal displacement of the platform, a and σ are the amplitude and frequency of platform heave, respectively, and K is the wave number in which, $K=2\pi/L_w$, L_w denote the wave length.

2.1. Equations of motion

Because the diameter of riser is much lesser than its length, so the Euler-Bernoulli is applicable for description of the pipe dynamic bending.

With above assumptions and according to the model in Fig.1, the equations of motion governing the in-line (IL) and cross-flow (CF) displacements of the riser as a function of depth z and time t can be written as

$$EI\frac{\partial^4 w}{\partial z^4}-\frac{\partial}{\partial z}\left[T_{e(z,t)}\frac{\partial w}{\partial z}\right]+m_t\frac{\partial^2 w}{\partial t^2}=F_{w,t(z,t)} \tag{4}$$

$$EI\frac{\partial^4 y}{\partial z^4}-\frac{\partial}{\partial z}\left[T_{e(z,t)}\frac{\partial y}{\partial z}\right]+m_t\frac{\partial^2 y}{\partial t^2}=F_{y,t(z,t)} \tag{5}$$

where EI is the riser bending stiffness, m_t is the total mass of the riser per unit length (riser with internal fluid and added mass), $F_{w,t(z,t)}$ and $F_{y,t(z,t)}$ are the IL and CF total fluid force per unit length, respectively that can be expressed as

$$F_{w,t(z,t)}=\frac{1}{2}\rho AC_D(v_w+u-\frac{\partial w}{\partial t})\left|v_w+u-\frac{\partial w}{\partial t}\right|+\rho VC_M\frac{D(u+v_w)}{Dt} \tag{6}$$

$$F_{y,t(z,t)}=\frac{1}{2}\rho DC_L\left(v_w+u-\frac{\partial w}{\partial t}\right)^2 Cos(\omega_s t)-\frac{1}{2}\rho AC_D\left(\frac{\partial y}{\partial t}\right)\left|\frac{\partial y}{\partial t}\right| \tag{7}$$

where ρ is the water density, A is the normal to current area of riser, C_D is the drag force coefficient, V is the volume of riser per unit length, C_M is the coefficient of inertia force, D is the outer diameter of the riser, C_L is the lift force coefficient, ω_s denote the shedding frequency of vortex, $D(v_w+u)/Dt$ is the acceleration of fluid in any depth, v_w and u are the wave velocity and current velocity in any depth, respectively.

$$v_W=a\sigma\frac{Cosh\,K(h-z)}{Sinh(Kh)}Cos\,(Kw-\sigma t) \tag{8}$$

$$u=v_t+v_z\left(1-\frac{z}{h}\right) \tag{9}$$

where h is the water depth, v_t and v_z are positive constant and v_t shows the fluid velocity on the bottom of sea.

The boundary conditions at the ends of the riser, assuming that the connection of the heave compensator to the riser can be modeled as a hinge, are given as

$$\begin{cases} w_{(l,t)}=0 \\ EI\left.\frac{\partial^2 w}{\partial z^2}\right|_{(l,t)}=0 \end{cases} \qquad \begin{cases} w_{(0,t)}=w_0 \\ EI\left.\frac{\partial^2 w}{\partial z^2}\right|_{(0,t)}=0 \end{cases} \tag{10}$$

$$\begin{cases} y_{(l,t)}=0 \\ EI\left.\frac{\partial^2 y}{\partial z^2}\right|_{(l,t)}=0 \end{cases} \qquad \begin{cases} y_{(0,t)}=0 \\ EI\left.\frac{\partial^2 y}{\partial z^2}\right|_{(0,t)}=0 \end{cases} \tag{11}$$

The values of the parameters that are used in this paper are shown in Table 1. It should be noted that the specifications from the Iran Alborz semi-submersible platform are used.

h	800 m	v_t	0.1 m/s	C_M	1.8
l	800 m	v_z	0.84 m/s	k_m	0.8
D	0.53 m	L_w	300 m	C_L	0.5
d	0.48 m	σ	0.67 rad/s	ω_s	1.07 rad/s
ρ_s	7850 Kg/m^3	a	3.5 m	ρ_f	2175 Kg/m^3
E	200 GPa	C_D	0.8	ρ	1025 Kg/m^3

Table 1. Values of system parameters used in calculations.

2.2. Static analysis

Before a dynamic analysis can be carried out, it is necessary to find the static equilibrium condition. The static analysis is carried out only for the IL configuration since it is the flow direction. Neglecting the time dependent terms in Eq. (4) and considering the floating platform in the lowest position, so the equation governing static equilibrium of the riser can be expressed as

$$EI\frac{d^4 w}{dz^4}+W_B\frac{dw}{dz}-\left\{W_B(-z+fl)+k\left[\left((l-a)^2+w_0^2\right)^{1/2}-l\right]\right\}\frac{d^2 w}{dz^2}$$
$$-\frac{1}{2}\rho DC_D\left[v_t+v_z\left(1-\frac{z}{h}\right)\right]^2=0 \tag{12}$$

Applying the Galerkin method, the general solution of this equation can be written as

$$w_{(z)}=C_1 Sin\left(\frac{\pi z}{l}\right)+C_2 Sin\left(\frac{3\pi z}{l}\right)+C_3 \tag{13}$$

Substituting Eq. (13) into the boundary conditions a system of linear algebraic equations can be obtained with respect to the unknown constants C_1-C_3. Fig. 2 shows the shape of the riser in the static equilibrium. The dynamic response is assessed about this stability deflected shape.

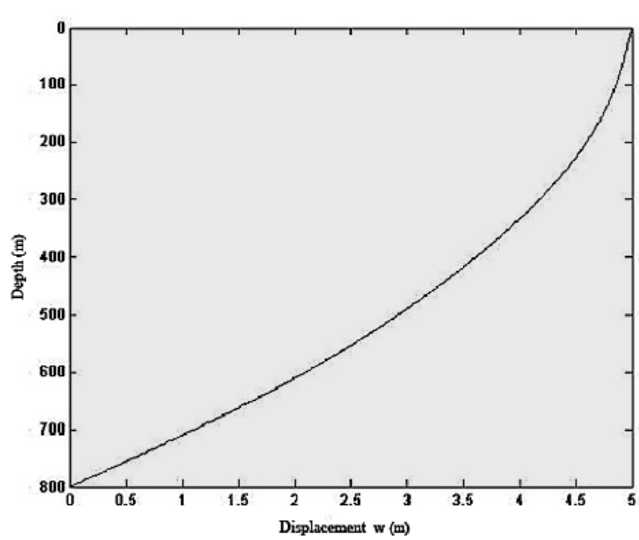

Fig. 2. Shape of the riser in the static equilibrium.

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3. Results and discussion

To study the dynamic equilibrium the original nonlinear equations of motion, Eq. (4) and Eq. (5), with the boundary conditions, Eq. (10) and Eq. (11), are solved using finite difference and Range-Kutta numerical methods. The following initial conditions are used

$$\left(\frac{\partial w}{\partial t}\right)_{t=0} = 0 \tag{14}$$

$$y_{(z,0)} = 0 \qquad \left(\frac{\partial y}{\partial t}\right)_{t=0} = 0 \tag{15}$$

It is assumed that the riser start to IL vibration of static equilibrium shape. Employing the above described procedure, Fig. 3 and Fig. 4 are obtained that are shown the IL and CF deflection along the riser in the dynamic equilibrium when the influence of initial conditions has almost disappeared. At first glance. It might seem remarkable that in these figures the areas near sea level experience a large displacement respect to other areas of the riser.

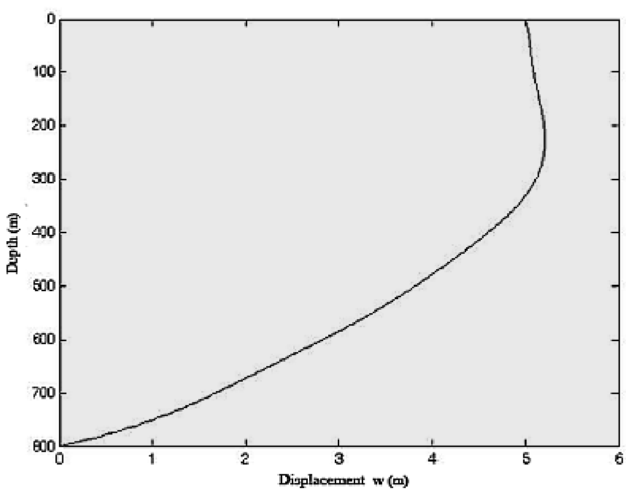

Fig. 3. Configuration of in-line displacement in the dynamic equilibrium

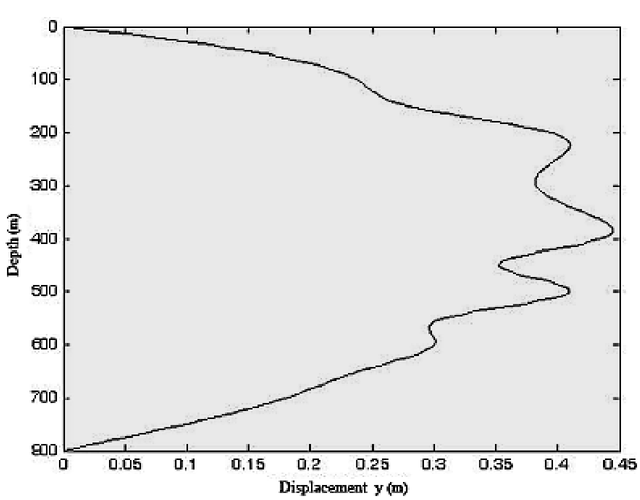

Fig. 4. Configuration of cross-flow displacement in the dynamic equilibrium

3.1. Dynamic analysis

Fig.5 and Fig.6 shows the effect of platform positions on the IL and CF response of the riser. When the platform is moving to the lowest position, t=193 s, the deflection of the risers increases near the sea bottom. At t=209, the platform is in the lowest position and the riser experiences the most IL and CF deflection. Indeed, the mean value of the tension of the riser decreases with the distance from the sea surface. Therefore, the platform, if heaving with sufficiently large amplitude, can change the tension into compression near the sea bottom. When the platform is moving up, t=223s, it can be seen that the displacement amplitude of the riser decreases but the deflection near the midpoint of the riser increases. Indeed the created pulses near the sea bottom in previous moment propagated along the riser. At t=240s, the platform is in the highest position and the IL and CF deflection along the riser experience remarkable decrease. In this position the tension has the highest value.

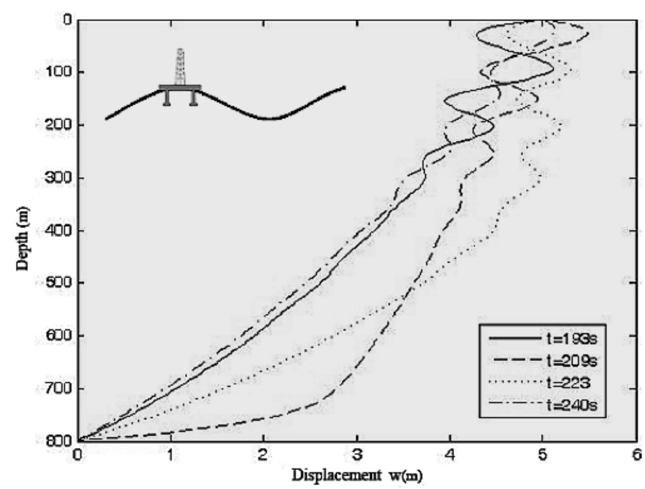

Fig. 5. Configuration of in-line displacement in four positions of platform.

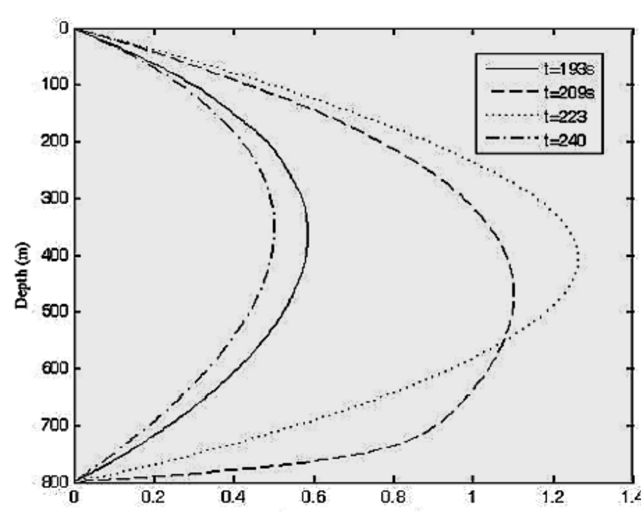

Fig. 6. Configuration of cross-flow displacement in four positions of platform.

Fig.7 and Fig.8 shows the influence of wave amplitude on the IL and CF deflection along the riser. It can be seen that increase in wave amplitude causes a significant increase in the deflection of the riser, especially near the seabed.

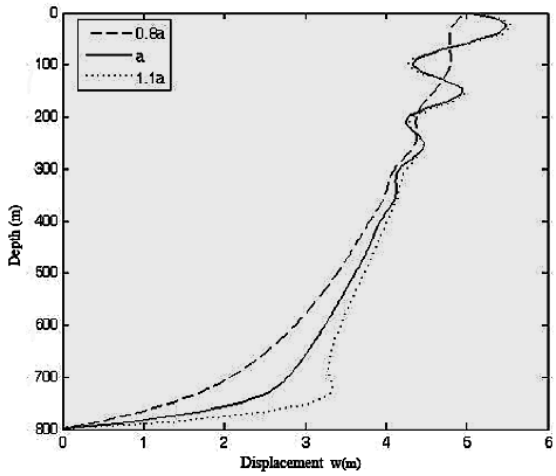

Fig. 7. Effect of wave amplitude on the in-line displacement.

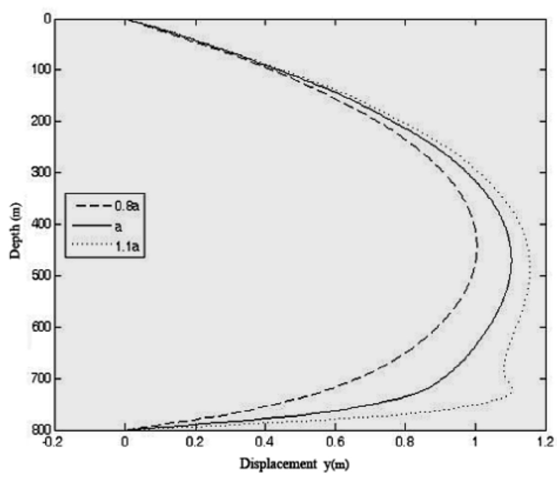

Fig. 8. Effect of wave amplitude on the cross-flow displacement.

The effect of compensator parameters are considered in Fig.9 and Fig.10. To study the effect of compensator, the platform is assumed at its lowest position.

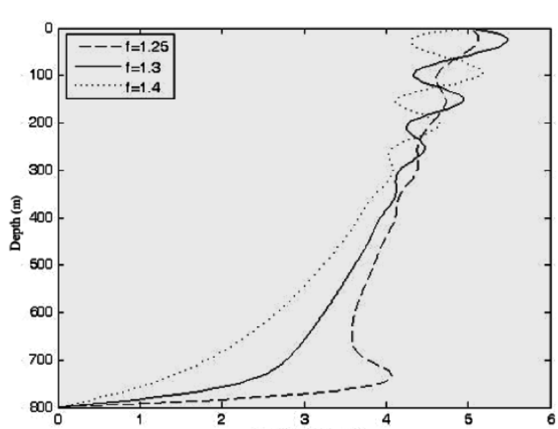

Fig. 9. Effect of pretension factor on the in-line displacement.

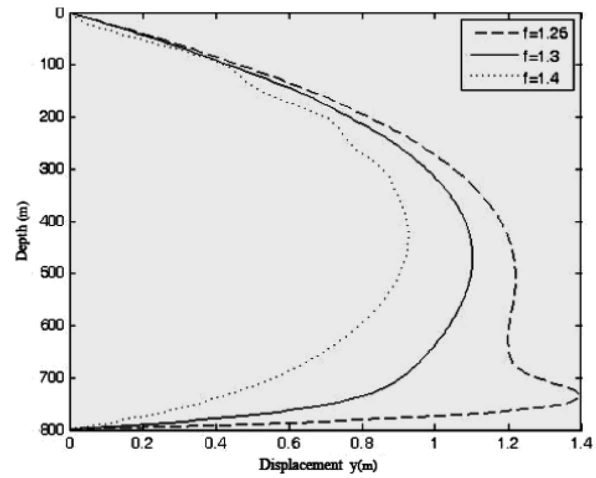

Fig. 10. Effect of pretension factor on the cross-flow displacement.

Fig.11 and Fig.12 show that the higher stiffness causes a significant growth in the IL and CF deflection along the riser. Indeed the increase in stiffness causes more dependency between the platform position and the riser.

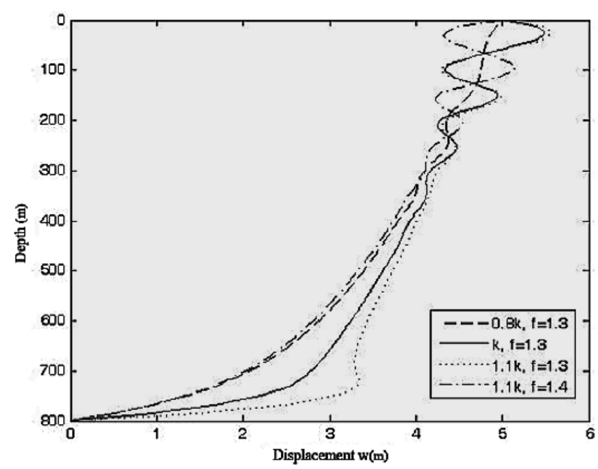

Fig. 11. Effect of compensator parameters on the in-line displacement.

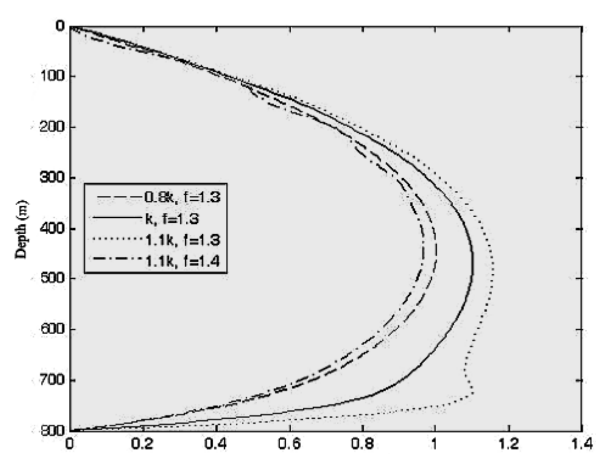

Fig. 12. Effect of compensator parameters on the cross-flow displacement.

It can be seen from these figures that if the pretension factor, *f*, increases, the vibration amplitude of the riser decreases. A comparison between the effects of the compensating parameters shows that the stiffness *k*, has a more significant influence on the IL and CF response of the riser. Therefore relatively soft heave compensators are recommended to ensure control of the riser behavior.

3.2. Effect of middle tension

It is obvious that the main role of top-tension is reducing the buckling effects of submerged weight near the seabed. So around the sea level, that the main force is caused by wave field, this tension can't have any significant influence on the deflection. Fig.13 shows the riser deflection when the platform

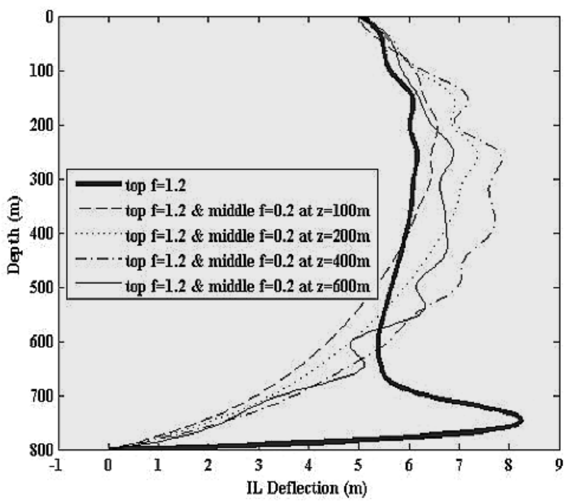

Fig. 13. Effect of middle tension on the In-Line displacement.

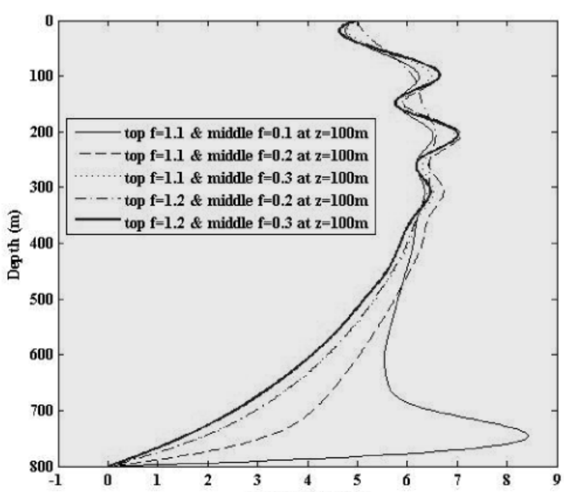

Fig. 14. Comparison between top-tension and middle tension on the In-Line displacement.

is in the lowest position. It can be found that using a small top-tension increases the displacement in the depth. But if it support with a fitting middle tension the deflection reduces whereas the axial tension is fixed in over area. Reducing the force level decrease the damage accumulated along the riser and improves the fatigue life. Also it can be seen the increase in middle tension depth leads to grow displacement. The main reason of this phenomenon is the creating and moving vibration pulses along the riser. With decrease in under tension length, these pulses create in the depth and traverse throughout the cylinder. If the pulses create and move with a sufficient speed, they can enrich each other and amplify displacement.

Fig.14 illustrates the effect of variation in top-tension and middle tension on the dynamic deflection. It can be obtained that increase in top-tension doesn't have significant effect on the deflection amplitude around the sea level whereas a slight rise in middle tension can reduce displacement significantly. It is obvious from these figures that the middle tension depth has a determiner effect on the dynamic deflection of the riser.

3.3. Natural frequencies and resonance

It is explained that the riser don't have constant tension along its length. However, to obtain natural frequencies the tension variation can be moderated and with a good approximation $\overline{T}$ can be set equal to the averaged tension that only is a function of platform position. Based on this assumption, the equation governing free vibration of the riser can be obtained as

$$EI\frac{\partial^4 w}{\partial z^4} - \overline{T}\frac{\partial^2 w}{\partial z^2} + m_t\frac{\partial^2 w}{\partial t^2} = 0 \tag{16}$$

$$\overline{T} = W_B l(f-0.5) + kaCos(\sigma t) \tag{17}$$

Solving Eq. (16), gives Eq.(18) as the natural frequencies of the riser.

$$\omega_n = \frac{\pi^2}{l^2}\sqrt{\frac{EI}{m_t}}\left[n^4 + \frac{n^2 l^2}{\pi^2 EI}\left[W_B l(f-0.5) + kaCos(\sigma t)\right]\right]^{1/2} \tag{18}$$

where *n* is the mode number.

It is important to note that the natural frequencies are dependent on the platform position and change with time. In other word the system is changing during the time. Fig.15 shows that the influence of the platform position on the natural frequencies increases in higher modes.

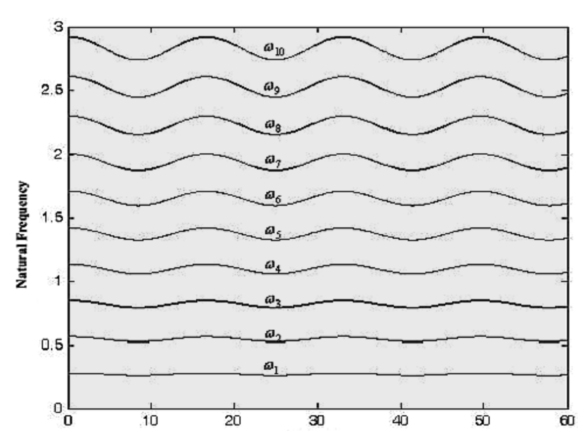

Fig. 15. Effect of platform position on the natural frequencies

To study the resonance phenomenon the CF response of the riser for the midpoint is considered. Fig.16 shows the deflection of the midpoint before resonance. The CF primary response, $\omega_s=\omega_1$ of the midpoint ($z=400m$) is shown in Fig.17. The comparison between Fig.16 and Fig.17 shows that in first parametric resonance the amplitude of deflections increases.

Fig.18 shows the composite resonance mode that occurs under combined wave-current, $\omega_s=\sigma=\omega_1$ so the amplitude of deflection increases greatly but the velocity of response decreases. Fig.19 illustrates the resonance mode what can be called super-harmonic resonance. It can be obtained that in this mode; the deflection of the riser has its greatest amplitude and least frequency.

This type of resonance occurs if the effective tension of the riser changes to compression near the seabed. In this time, the riser deflection grows locally and propagates along the riser. Then, the platform moves on the wave and the compression changes to tension again. Because of the low frequency of these deflections, they can't amplify each other but each disturbance travels along the riser until vanishing due to hydrodynamic damping. In this mode the distance that can be traveled by pulses is more dependent on the wave amplitude than frequency.

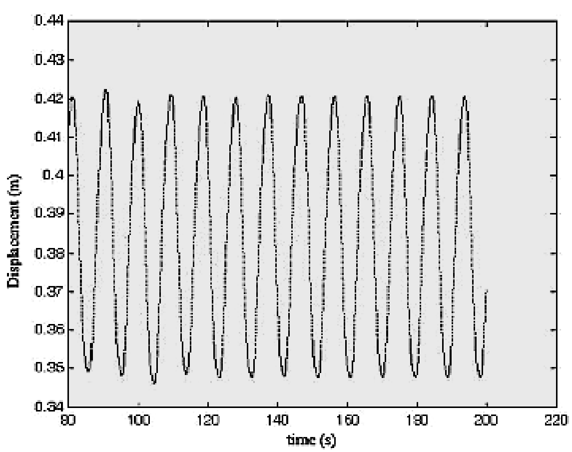

Fig. 16. Cross-flow deflection of the midpoint of the riser before resonance.

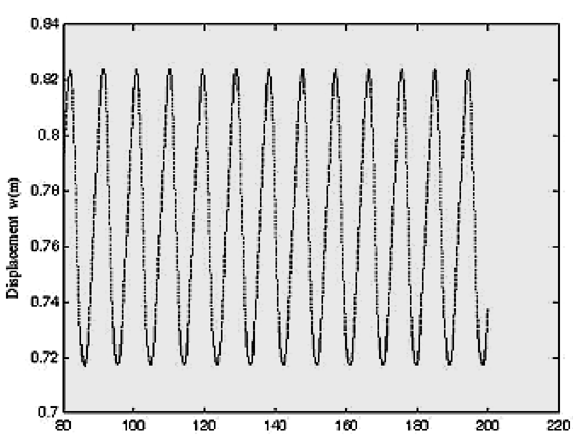

Fig. 17. Cross-flow deflection of the midpoint of the riser for parametric resonance.

Fig.20 shows the last resonance that is considered in this study what can be called sub-harmonic resonance.

This figure clearly shows that in this resonance mode the amplitude of deflections is similar to parametric resonance mode but the frequency of pulses increases. This behavior is reached via periodic generation of pulses near the seabed. In

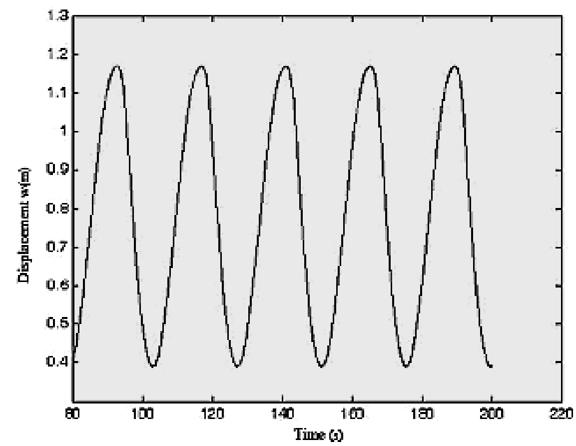

Fig. 18. Cross-flow deflection of the midpoint of the riser for composite resonance, $\omega_s=\sigma=\omega_1$.

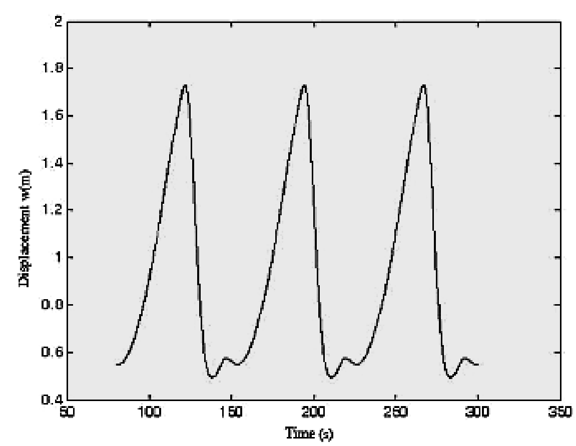

Fig. 19. Cross-flow deflection of the midpoint of the riser for super-harmonic resonance.

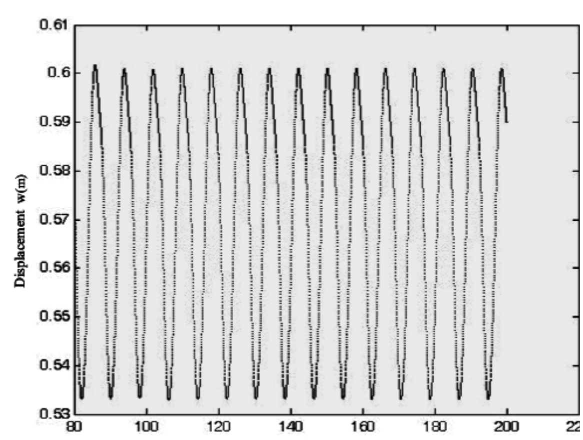

Fig. 20. Cross-flow deflection of the midpoint of the riser for sub-harmonic resonance,

this mode each new pulse is generated before the previous ones have been dissipated by the hydrodynamic damping so the frequency of pulses is higher than super-harmonic and parametric resonance modes.

4. Conclusions

In this paper a mathematical model of IL and CF vibration is suggested for the riser in deep waters and the dynamic responses of the riser are obtained in two plans. The effect of variation in mathematical and environmental parameters on the dynamic behavior is considered. Also the resonance modes of the riser are studied and the mechanisms of sub-harmonic and super-harmonic resonance are investigated. By processing the results data, the following conclusions can be drawn:

1. Around the sea level, the wave field dominates dynamic behavior and creates deflection pulses. These pulses don't have enough energy to travel along the riser and vanish before to get each other.

2. Reduction in axial tension along the riser due to platform movement or submerged weight caused the vibrating pulses creating and transverse near the sea bed. These pulses can move along the riser until disappear due to water damping effects.

3. Middle tension can prevent the creating of high amplitude deflection and moving them. So it controls the dynamic behavior of riser without imposing extra tension to upper bound of riser.

4. If vibrating pulses create and move with a sufficient speed, they can enrich each other and amplify displacement. This phenomenon appears when the resonance modes are occurred.

5. If the mechanical parameters, k, f, determine inexactly, the effects of platform motion on the dynamic behavior of riser increase intensively and can cause large deflection especially around the wellhead.

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