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Towards the Optimal Solution of Feeder Container Ships Routing With Empty Container Repositioning

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ABSTRACT

This paper presents a designing method to route a feeder container ship within a hub-and-spoke network that incorporates empty containers repositioning among the ports that are called. It enables to determine the sequence of calling ports as well as the number of full and empty containers transported between any two calling ports. The method is based on a Mixed Integer Linear Programming (MILP) formulation which enables to find optimal transport routes of feeder container ships, i.e. routes that maximize the profit of a shipping company. Our MILP formulation is based on a Knapsack problem and is converted to a location routing problem. The MILP model is tackled by the commercial CPLEX MIP solver. The results of the analysis show that the model can support the decision making process of a shipping company in establishing container feeder transport services. Proposed MILP models can be adapted, by simple changes, to various practical cases.

1. Introduction

Deep sea container shipping is a very competitive business, in which shipping lines compete with each other above all on freight rates. In transporting containers at sea it is difficult for shipping lines to distinguish themselves from competitors on other aspects than the rate level. This makes customers very sensitive to price differences between shipping lines and willing to switch easily from one shipping line to another. As a result the market characteristics force shipping lines to quote sharp freight rates and hence it is evident that controlling costs is of major importance to shipping lines. Shipping lines are keen on ways to control cost to have low cost operations both in exploitation of vessels and container management.

Increasing scale of operations has been a successful strategy to improve competitiveness by exploiting economies of scale. The emerge of ever increasing vessels has been a driving force for the introduction of the hub-and-spoke (HS) service network. In this network the large vessels can become more productive if they are operated in long-distance transport in corridors with large container flows and have a limited number of ports to visit, i.e. sailing in trunklines between mainports (hubs). In such a network, however, feeder services are needed between the hub and destination or origin ports of the containers, i.e. the spokes of the network. The rationale of the HS-system is that the costs of additional handling and the organisation of feeder services are offset by cost savings that arise from economies of scale. A good performance of the feeder service network is therefore a key factor for the success of the HS-system and hence for the profitability and competitiveness of the shipping line. In establishing a successful feeder service network the major decisions to be made include the size of vessels to operate in the network as well as their schedule in visiting seaports.

In order to control the total operational costs of container shipping, equipment management including container repositioning is also an activity that is of great importance to shipping lines (see e.g. Hultén, 1997). De Brito and Konings (2007) report that equipment repositioning accounts for, on average, 20 – 25 percent of the total operational costs of container shipping. The need to reposition containers is caused by cargo imbalances but also by a mismatch in type of equipment that is demanded (e.g. 20 ft containers) and available (e.g. 40 ft containers).

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Shipping lines have several strategies to control and reduce the costs of empty container transport (Konings and Thijs, 2001; Theofanis and Boile, 2008). Some strategies relate to pricing. For instance, shippers can be offered lower rates to use types of equipment that are in surplus. Alternatively, a freight rate surcharge can be imposed on the high demand leg to compensate the repositioning costs in the order direction. Other strategies rather focus on the organisation of container repositioning, such as the interchange of equipment between shipping lines. Concerning organisational strategies (Blanco, Pérez-Labajos, Sánchez, Serrano, López, Ortega, 2010) the design of the service network of the shipping line will also influence the costs of container repositioning. Although empty containers can be more easily repositioned in a larger network, because more seaports are connected with each other, its cost efficiency, however, depends on how well the ports are interconnected.

Since the characteristics of the whole service network of the shipping line are important in controlling container repositioning costs it is evident that the organisation of the feeder service network will also affect the costs of container repositioning. On the other hand the empty container repositioning process is likely to play a major role in the cost performance of feeder container services.

In this paper we will deal with these issues of controlling the costs of feeder container services and the costs of empty container transport. We present a designing method to route a feeder container ship within a HS-network that incorporates empty container repositioning among the ports that are called. This problem consists of determining the calling sequence as well as the numbers of full and empty containers transported between any two ports. The method is based on a Mixed Integer Linear Programming (MILP) formulation for the optimization of transport routes of feeder container ships with the objective to maximize the profit of a shipping company. The rest of this paper is organized as follows. In the next section we present a brief literature review. The description of a feeder container ship routing problem is given in the following section. Next, we propose a mathematical formulation of the problem and discuss the problem complexity. Experimental evaluations and final remarks conclude our study.

2. Literature review

Ship routing problems have strongly triggered research, especially during last three or four decades. In most studies, the results are related to deterministic cases of the ship routing problem, meaning that all input data and assumptions needed to optimise routing of ships were known in advance. On the other hand, relevant routing information (fuel price, container flows, etc) may change after the route has been established and the ship has started its journey. Routing problems that take such changes into account belong to the class of dynamic ship routing problems and are solved by using dynamic programming techniques. We briefly describe here some of the most relevant developments in this field.

Rana and Vickson (1988) proposed a mathematical programming formulation to evaluate chartering options for container ships. The model is developed for the calculation of the optimal sequence of calling ports, the number of containers transported between each pair of ports, and the number of trips the ship makes in the chartered period. Rana and Vickson (1991) extended the model presented in Rana and Vickson (1988) for routing multiple ships. Cho and Perakis (1996) presented routing and fleet design models for a container shipping company. Bendall and Stent (2001) developed a mixed integer program to determine the optimal fleet size and the profitability of a short-haul hub and spoke feeder operation based in Singapore. Castells and Martinez de Osés (2006) studied and identified the feasible short sea shipping routes in SW Europe which appear to be viable solutions for avoiding road transportation problems such as traffic congestion and high fuel consumption, implying the pollution and safety problems.

Shintani at el. (2007) addressed the issue of designing service networks for container liner shipping in deep sea transport, while explicitly taking into account empty container repositioning. A genetic algorithms (GA) based heuristic was proposed to find a set of calling ports, an associated port calling sequence, the number of ships (by ship size category) and the resulting cruising speed to be deployed in the service networks, with the objective to maximise the profit of a liner shipping company. This approach was applied to a case-study on container transportation in Southeast Asia.

Andersen (2010) considered the problem of designing the service network and schedule for a container shipping line in feeder operations with the objective to minimize the cost of operating this network. Two key planning problems, faced by container feeder service providers, were identified: 1) tactical service network design including fleet size and composition of the fleet, together referred to "the master schedule problem" and 2) definition of the sailing schedule and its implementation. Computational experiments showed that the presented algorithms were capable to solve real-world problems on both tactical and strategic planning levels within a reasonable time frame.

Yang and Chen (2010) studied the effects of changing an existing container shipping network that covered several regions in two countries, into a shipping network that consists of trunk and feeder lines on exploitation costs. Jetlund and Karimi (2004) addressed the scheduling of multi-parcel chemical carriers engaged in the transportation of multiple chemicals. The authors developed a MILP formulation for one-ship and multi-ship problems, and implemented a good heuristic based on the one-ship model. Similar work can be found in Hwang (2005) and Al-Khayyal i Hwang (2007). Erera et al. (2005) and Karimi et al. (2005) considered the decision problems (operational tank container management problem, scheduling of the transport and cleaning of multiproduct tank containers) faced by tank container operators.

We propose a new approach to route a feeder container ship within a hub-and-spoke network based on MILP formulation. It takes into account empty containers repositioning, and aims to maximize the profit of a container shipping line in feeder operations.

3. A feeder container ship routing problem

We address a feeder container ship routing problem in which the aim is to maximize the shipping company profit while picking up and delivering containers and taking into account empty container repositioning. Feeder container services, as well as other liner container routes, are usually organized as weekly services. These services are offered for relatively long periods of time, i.e. from several months to several years. Hence the decision to establish a feeder container service has to take into account different factors such as seasonal fluctuations of container flows throughout a year, market developments, etc. Shipping companies estimate the demand for container transport in any potential port of call and these data strongly influence the decision on the routing of feeder container ships. These estimations are usually made on a weekly basis, together with estimates on costs and revenues related to these demand forecasts for the same periods. In addition, a lot of other parameters such as ship speed, ship size, and number of available ships influence the routing decisions of feeder container ships.

Shipping companies assign their ships to specific routes which include more than two ports of call. Transshipment or hub ports are particularly important in this process. These ports are the links between trunkline and feeder container ship routes. Therefore, hub ports are starting and ending ports of call in the feeder service. Regular feeder route links the hub port with few smaller, feeder ports located in the catchment region of that particular hub port. This means that a feeder ship starts its route in the hub port, visits several feeder ports and ends the route by coming back to the same hub port. Containers that are loaded on a feeder ship in one port are usually transported to several destination ports. Schedules of feeder container routes should be known in advance, with arrival and departure times indicated for each port of call.

The following assumptions are made in our model regarding the routing of feeder container ships within container transport networks:

— The routing of feeder ships is considered for one geographical region which includes a number of feeder container ports that are connected to one transshipment or hub container port;
— It is not necessary for the feeder container ship to visit all ports in the region; in some cases, calling at a particular port or loading all containers available in that port may not be profitable due to high port fees, costs of handling containers and/or because the quantity of containers that has to be transported to or from this port is too small;
— The ship can call in one port more than once;
— The starting and ending point on the route should be the same, i.e. in this case it is the hub port where the transshipment of containers from trunkline to feeder container ships and vice versa takes place;
— The shipping company that provides feeder services has to bear transport costs of loaded containers as well as empty container repositioning costs, transhipment costs and port fees;
— The model assumes a weekly known cargo demand for each pair of ports; this assumption is valid as the data regarding throughput from previous periods and future prediction enable to obtain reliable values of these demands;
— Container volume loaded at a port may be less than the number of containers generated in that port; some containers may not be profitable for the feeder container company, primarily because of the small number of containers, low freight rates, destination port of the containers in the considered region not being included in the route, etc.;
— Total number of loaded and empty containers onboard may not exceed the ship carrying capacity at any link of the route;
— The demand for empty containers at a port is the difference between the total traffic originating from the port and the total number of loaded containers arriving at the port for the specified time period; the assumption is valid since this study addresses the problem of determining the optimal route of a feeder container ship for only one ship operator (similar to the case studied by Shintani et al., 2007);
— If a sufficient container quantity is not available at a port, the shortage is made up by leasing containers with the assumption that there are enough containers to be leased (for details see Shintani et al., 2007).

Under those assumptions, we model the following two decisions of the shipping company in operating the feeder network:

1. The number of ports to call and the calling sequence in the route,
2. The size of container batches to be transported between any two ports in the route.

Ports of call, calling sequence and container flows are the factors that directly influence revenue and costs, and hence the profitability of the feeder container service.

The following data (measurement units are given in square brackets if applicable) are included in the model:

L: number of ports;
v: container ship cruising speed, [kts];
R_f and R_l: fuel and lubricant consumption, respectively [t/kWh];
C_f and C_l: fuel and lubricant price, respectively [US$/t];
C_{TEU}: carrying capacity of container ship [TEU];

D: ship displacement [t];
A: Admiralty coefficient;
P_{out}: engine output (propulsion) [kW];
dcc: daily time charter cost of container ship [US$/day];
max_{tt} and min_{tt}: maximum and minimum turnaround time on a route [days];
$zr_{(i,j)}$: weekly expected number of loaded containers available to be transported between ports i and j [TEU];
$r_{(i,j)}$: freight rate per container from port i to port j [US$/TEU];
$l_{(i,j)}$: cruising distance from ports i to j [NM];
ufc_i and lfc_i: unloading and loading cost, respectively per loaded container at port i [US$/TEU];
uec_i and lec_i: unloading and loading cost , respectively per empty container at port i [US$/TEU];
pec_i: entry cost per call at port i [$];
uft_i and lft_i: average unloading and loading time, respectively, per loaded container at port i [h/TEU];
uet_i and let_i: average unloading and loading time, respectively, per empty container at port i [h/TEU];
pat_i and pdt_i: standby time for arrival and departure, respectively, at port i [h];
sc_i: storage cost at port i [US$/TEU/day];
lc_i: short-term leasing cost at port i [$/TEU/day].

4. Problem formulation

This section describes a mathematical programming formulation, in a form of mixed integer linear programming, for a feeder container ship routing problem. The goal is to determine the optimal feeder container route, i.e. the selection of the optimal set of calling ports and sequence of calls, as well as the size of container batches to be transported between any two ports in the route.

At the beginning, it is necessary to introduce a parameter k that refers to the number of segments in the container ship route. Each segment represents a sailing between two consecutive ports (i and j) in the route. Since a feeder container ship should return to the starting (hub) port at the end of its route, the minimal number of segments is 2. On the other hand, the total number of segments K that will be considered should be determined by decision makers of a container shipping line in feeder operations. More precisely, if K denotes the total number segments in the route, than K = 2, 3, ..., n. Each segment in the route is denoted by k = 1, 2, ..., K. Moreover, the navigation on each route segment is characterized by travel time and number of full and empty containers carried onboard. Consequently, the variables in the model are as follows:

x_{ik}: 1 if ship arrives at port i at the end of segment k, 0 otherwise;
q_{ijk}: 1 if ship moves from port i to j along segment k, 0 otherwise;
$pl_{(i,j),k}$: 1 if ship loads cargo (i, j) at the end of segment k, 0 otherwise;
$pd_{(i,j),k}$: 1 if ship unloads cargo (i, j) at the end of segment k, 0 otherwise;
$sb_{(i,j),k}$: 1 if ship carries cargo (i, j) on-board during segment k, 0 otherwise;
$y_{(i,j)}$: 1 if ship serves cargo (i, j), 0 otherwise;
t_k: ship voyage time on a segment k;
t_s: total turnaround time [h];
z_k, w_k: the number of loaded and empty containers, respectively, transported along a segment k [TEU];
a_k: starting port of segment k;
Y^K: shipping company profit for the route consisting of K segments.

Optimal feeder container ship route consisting of K segments is modelled as follows:

$$\text{maximize} \quad Y^K \tag{1}$$

s.t.

$$\sum_{i=1}^{L} x_{ik} = 1, \quad \forall k = 1,\dots,K \tag{2}$$

$$\sum_{j} q_{jik} = x_{ik}, \quad \forall k,\ \forall i \tag{3}$$

$$\sum_{j} q_{ij(k+1)} = x_{ik}, \quad \forall k < K \tag{4}$$

$$\sum_{j} q_{ijk} = 1, \quad i = 1 \text{ and } k = 1 \tag{5}$$

$$\sum_{j} q_{jik} = 1, \quad i = 1 \text{ and } k = K \tag{6}$$

$$\sum_{k=1}^{K} pl_{(i,j),k} = y_{(i,j)}, \quad \forall i, j \tag{7}$$

$$\sum_{k=1}^{K} pd_{(i,j),k} = y_{(i,j)}, \quad \forall i, j \tag{8}$$

$$pl_{(i,j),k} \le x_{i,k}, \quad \forall i, j \text{ and } \forall k \tag{9}$$

$$pd_{(i,j),k} \le x_{j,k}, \quad \forall i, j \text{ and } \forall k \tag{10}$$

$$sb_{(i,j),k+1} = sb_{(i,j),k} + pl_{(i,j),k} - pd_{(i,j),k}, \quad \forall i, j \text{ and } \forall k \tag{11}$$

where: if $k = K \Rightarrow k+1$ is replaced with $k = 1$

$$sb_{(i,j),k} \le y_{(i,j)}, \quad \forall i, j \text{ and } \forall k \tag{12}$$

$$z_k + w_k \le C_{TEU}, \quad \forall k \tag{13}$$

$$\min_{tt} \le \frac{t_s}{24} \le \max_{tt}, \tag{14}$$

The round–trip time is calculated as the sum of voyage time, handling time of loaded and empty containers in ports and time required for entering/leaving ports (15).

$$t_s = \frac{1}{24}\left(\frac{\sum_i \sum_j l_{ij} q_{ijk}}{v} + \sum_{i=1}^{L}\sum_{j=1}^{L} zr_{(i,j)} y_{(i,j)} \left(lft_i + uft_j\right) + \sum_{i=1}^{L}\sum_{j=1}^{L} w_{ij} \left(let_i + uet_j\right) + \sum_{i=1}^{L}\sum_{k=1}^{K} x_{ik}\left(pat_i + pdt_i\right) \right) \tag{15}$$

To determine the amount of the revenues and costs, it is necessary to incorporate the appropriate parameters and variables describing the feeder shipping service under consideration. So, on its route, a feeder container ship can call at any port from the set of given ports L (i = 1, 2, ..., L). It is assumed that the port i = 1 represents a transhipment (hub) port, while all other ports, j = 2, 3, ..., L, are feeder container ports in the catchment region.

Constraints (2) force the feeder container ship to visit exactly one port during each segment. Constraints (3) – (6) are network constraints ensuring that the ship starts and ends the route at the transshipment port making a connected trip. The ship is left with a choice of calling or not calling at any port. If the ship is assumed to load a cargo (i, j) at the end of segment k, two conditions should be satisfied. First, the ship must visit the loading port and second, it must service cargo (i, j). Eqs. (7) and (8) enforce that if the ship does not service a cargo (i, j) (i.e. $y_{(i,j)} = 0$), then it cannot load (unload) it at any port. However, $y_{(i,j)} = 1$ forces loading (unloading) of cargo (i, j) at exactly one segment. Constraints (9) and (10) are to ensure that if the ship does not visit the pickup (delivery) port of a cargo (i, j) at the end of segment k, then it cannot pickup (deliver) cargo (i, j) at the end of segment k. Eqs. (11) are applied to force the ship to carry a cargo (i, j) from its pickup port to its discharge port.

Determining a transport route of a feeder container ship belongs to the class of the so called long-term problems. This means that operated period of established routes range from several months to several years and involve a large number of round trips. Therefore, the transportation process should include containers loaded during one round trip and unloaded during the next one (constraints (11) and (12)).

Capacity constraints (13) guarantee that the total number of full and empty containers on-board does not exceed the ship carrying capacity during each sailing segment. Constraint (14) is related to the round–trip time of the feeder container ship (denoted by t_s[h]). This constraint has to prevent the feeder ship ending and calling at port 1 long before or after the arrival of the trunkline container ship.

Methods of determining the total number of both full, z_k, and empty, w_k, containers on-board at any voyage segment k, given by Eq. (13), require further explanations. While calculation of full containers transported along any segment is straightforward, the determination of the number of empty containers on-board is much more complex task. The differences come from the fact that expected weekly containers demand (i, j) to be transported between ports i and j is known in advance. Therefore, the total number of full containers transported along any voyage segment is given as follows:

$$\sum_{i=1}^{L}\sum_{j=1}^{L} zr_{(i,j)} sb_{(i,j),k} = z_k \quad \forall k \tag{16}$$

On the other hand, quantity of empty containers that can be transported depends on the available capacity of the ship for empty containers. In order to deal with empty containers repositioning, we introduce the following auxiliary variables: the number of empty containers to be transported between ports i and j, w_{ij}, the number of containers to be stored at each port i, sW_i, and the number of containers to be leased at each port i, lW_i (Shintani et al., 2007). After the linearization we obtain constraints (17)-(24):

$$S_i - M \cdot g_i \le 0 \tag{17}$$

$$D_i - P_i + S_i \ge 0 \tag{18}$$

$$D_i - P_i + S_i - M(1 - g_i) \le 0 \tag{19}$$

$$Q_i - M \cdot h_i \le 0 \tag{20}$$

$$D_i - P_i + Q_i \ge 0 \tag{21}$$

$$D_i - P_i + Q_i - M(1 - h_i) \le 0 \tag{22}$$

$$lW_i = Q_i - \sum_{j=1}^{L} w_{ji} \quad \forall i \in 1,\dots,L \tag{23}$$

$$sW_i = S_i - \sum_{j=1}^{L} w_{ij} \quad \forall i \in 1,\dots,L \tag{24}$$

where:

- g_i, h_i: auxiliary binary variables, $\forall i \in 1,\dots,L$
- Q_i: the number of demanded containers at each port i [TEU];
- S_i: the number of excess containers at each port i [TEU];
- P_i: the number of containers destined for port i [TEU];
- D_i: the number of containers departing from port i [TEU];
- M: large enough constant.

As it can be seen, empty container flows are defined between any two ports i and j, rather than for each voyage segment k. In order to calculate the total number of empty containers (w_k) transported along segment k, it is necessary to determine the starting port of segment k, denoted by a_k:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} i \cdot q_{ijk} = a_k, \quad \forall k \tag{25}$$

Now, w_k can be calculated as:

$$w_k = \begin{cases} \sum_{i=1}^{k}\sum_{j=k+1}^{n} w_{a_i,a_j} + \sum_{i=k+2}^{n}\sum_{j=k+1}^{i-1} w_{a_i,a_j}, & k = 1 \\ \sum_{i=2}^{k}\sum_{j=1}^{i-1} w_{a_i,a_j} + \sum_{i=1}^{k}\sum_{j=k+1}^{n} w_{a_i,a_j} + \sum_{i=k+2}^{n}\sum_{j=k+1}^{i-1} w_{a_i,a_j}, & 1 < k < K-1 \\ \sum_{i=2}^{k}\sum_{j=1}^{i-1} w_{a_i,a_j} + \sum_{i=1}^{k}\sum_{j=k+1}^{n} w_{a_i,a_j} & k = K-1 \\ \sum_{i=2}^{k}\sum_{j=1}^{i-1} w_{a_i,a_j} & k = K \end{cases} \tag{26}$$

Finally, the profit of the shipping company, Y^K, is calculated as the difference between the revenue arising from the service of loaded containers (R_K) and the transport costs. These costs are related to shipping (C_K) as well as empty container handling (E_K) (adopted from Shintani et al., 2007). More precisely:

$$Y^K = R_K - C_K - E_K \tag{27}$$

where:

$$C_K = (6{,}54 \cdot C_{TEU} + 1422{,}52) \cdot t_s + \sum_{k=1}^{K} \frac{(C_f R_f + C_l R_l) \cdot D^{\frac{2}{3}} \cdot v^3 \cdot 0{,}7457 \cdot t_k}{A} + \\ + \sum_{i=1}^{L} \sum_{k=1}^{K} pec_i C_{TEU} x_{ik} + \sum_{i=1}^{L} \sum_{j=1}^{L} zr_{(i,j)} y_{(i,j)} (lfc_i + ufc_j) \tag{28}$$

$$E_K = \sum_{i=1}^{n} (sc_i \cdot sW_i + lc_i \cdot lW_i) + \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} (uec_i + lec_j) \tag{29}$$

$$R_K = \sum_{i=1}^{L} \sum_{j=1}^{L} zr_{ij} y_{ij} p_{ij} \tag{30}$$

The above described model determines the optimal feeder container route for a given number of segments K. In order to determine the optimal number of segments, one should solve the related Knapsack problem, i.e.:

$$\text{maximize} \quad \sum_K Y^K p_K \tag{31}$$

s.t.

$$\sum_K p_K = 1 \tag{32}$$

$$p_K \in \{0,1\}, \quad \forall K = 2,3,\ldots,n \tag{33}$$

where p_K takes value 1 if the route with K segments is selected, 0 otherwise.

Problem complexity and optimal solution

Our problem can be categorized as a bi-level combinatorial optimization problem. The upper level of our ship routing problem, which selects the best set of calling ports, reduces to the Knapsack problem (well known to be NP-complete, Papadimitriou and Steiglitz, 1982).

On the other hand, the lower level model represents a mixture of several modifications of the standard ship routing problem. As container shipments are smaller than a feeder container ship capacity, our problem is a multi-commodity ship routing problem. Container loading and unloading processes make it a ship routing problem with pickups and deliveries. This problem is also a ship routing problem with time windows as we have limits on round trip time. Therefore, the feeder container ship routing problem considered here can be classified as a multi-commodity Pickup and Delivery Problem with Time Windows. It is an extension of the Vehicle Routing Problem, which in turn is a generalization of the NP-complete Travelling Salesman Problem. Consequently, our problem is strongly NP-complete.

All this implies that generally the problem cannot be solved in polynomial time. However, the examples considered here are not too complex. Therefore, our MILP formulation for this problem used within CPLEX MIP solver enabled to obtain optimal solutions for given instances. The conduced experimental evaluation is described in the next section.

5. Experimental evaluation

5.1. Input data

Due to the luck of publicly available test instances for the considered problem, we generated random test examples. We assume the number of ports equals 7, i.e. $L = 7$. The transport network that is analysed is given in the Figure 1.

Transport distances, expected number of loaded containers weekly available to be transported between any two ports and freight rates per container from port i to port j are derived from practice and are available upon request by the authors. This also applies to port related parameters which are divided in two groups: cost and time related parameters.

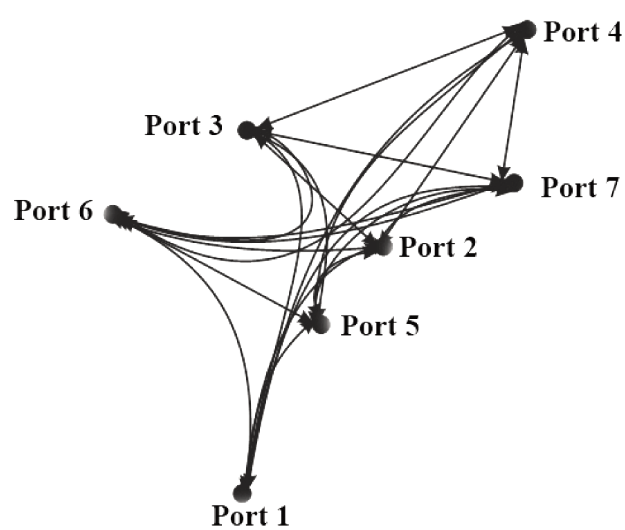

Figure 1. Transport network for routing a feeder container ship.

Settings of other parameters are as follows: C_{TEU} = 750 TEU; D = 13095 t; v = 15 kts; frequency: weekly; C_f = 170 \$/t; C_l: 1000 \$/t; R_f: 140 g/kW/h (0.00014 t/kW/h); R_l: 4 g/kW/h (0.000004 t/kW/h), $\max_{tt}$ = 14 days, $\min_{tt}$ = 13 days.

5.2. Numerical results

Our tests are performed on Intel Core 2 Duo CPU E6750 on 2.66GHz with RAM=8GB under Linux Slackware 12, Kernel: 6.21.5. For exact solving we used CPLEX 11.2 IP solver and AMPL (Fourer et al., 1990, ILOG, 2008) for Linux operating system and compiled with gcc (version.1.2) and the option -o2.

In practice, the maximum number of segments in the feeder container ship route is based on planners' experience. This decision has to take into account parameters like distances between ports, feeder container ship speed, weekly container demands, etc. Therefore, we impose that

Table 1. Results for 750 TEU ship and any number of segments.

Ship speed (knots)	Number of segments (*K*)	Calling sequence	Profit [$]	Total revenue [$]	Total costs [$]	Fleet size
15	5	1 – 7 – 4 – 3 – 6 – 1	18695,4	405420,0	386724,6	2
	6	1 – 2 – 7 – 4 – 3 – 6 – 1	38673,1	462770,0	424096,9	2
	7	1 – 3 – 2 – 7 – 4 – 3 – 5 – 1	36904,1	490885,0	453980,9	2
	8	1 – 5 – 3 – 2 – 7 – 4 – 3 – 5 – 1	39867,7	526230,0	486362,3	2
	9	1 – 5 – 3 – 2 – 7 – 4 – 2 – 3 – 5 – 1	27087,4	487860,0	460772,6	2

the maximal number of segments in the route equals 9. This means that we have to elaborate the cases where the number of segments goes from 2 to 9, i.e. $K = 2, 3, ..., 9$. Profit, Y^K, is calculated for each K and the optimal number of segments in the route is determined as K corresponding to the highest profit.

Results indicate that cases for $K = 2$ or $K = 3$ are not feasible (cannot generate a route with turnaround time longer than min_{tt}). If $K = 4$ the optimal solution generates a loss. Results of all the other cases ($K = 5,..., 9$) are given in Table 1.

By applying eqs. (31) – (33) the optimal solution is obtained for $K = 8$. Achieved profit, Y^8, in this case equals 39867,72 $/round trip. Total round trip time for $K = 8$ is 13,981 days.

among available sailing scenarios. The cases when the transport demands change are covered by the application of the same analysis to different input data.

The analysis evaluates the effects of chartering each of the three ships whose characteristics are summarised in Table 2 (taken from Shintani et al., 2007).

Table 2. Basic ships characteristics.

Type of ships	Carrying capacity [dwt]	Displacement [t]	Engine power [kW]	Speed intervals [knots]	Admiralty coefficient
500 TEU	7050	9517,5	4050	9-15	250
750 TEU	9700	13095	5500	11-17	275
1000 TEU	13500	18225	7600	13-19	300

5.3. Sensitivity analysis

This sensitivity analysis examines the impacts of two types of factors on feeder container ship routing decisions: container ship carrying capacity and sailing speed. Given the transport demands, we have to determine the optimal size of feeder container ship and its optimal sailing speed

The optimal routes for each ship and speed are determined by changing the number of segments from 4 to 9. As in Table 1 we omit the cases with non-feasible or negative results. The results that are obtained for the selected types of ships and corresponding speed intervals are given in Tables 3, 4 and 5 (for 500, 750 and 1000 TEU ships, respectively).

Table 3. Results for 500 TEU container ship.

Ship speed (knots)	Number of segments (*K*)	Route	Profit [$]	Revenue [$]	Total costs [$]
9	4	1 – 7 – 2 – 5 – 1	16001,5	247765,0	231763,5
	5	1 – 2 – 7 – 2 – 5 – 1	18292,2	271360,0	253067,8
	6	1 – 3 – 2 – 7 – 3 – 2 – 1	9860,4	258910,0	249049,6
10	4	1 – 7 – 4 – 3 – 1	18216,9	259890,0	241673,1
	5	1 – 5 – 7 – 4 – 3 – 1	19885,9	273615,0	253729,1
	6	1 – 5 – 3 – 7 – 2 – 5 – 1	20134,0	311715,0	291581,0
	7	1 – 5 – 3 – 7 – 2 – 3 – 5 – 1	16124,3	311715,0	295590,7
	8	1 – 5 – 3 – 2 – 7 – 2 – 3 – 5 – 1	5197,1	271360,0	266162,9
11	4	1 – 2 – 4 – 6 – 1	11442,3	246060,0	234617,7
	5	1 – 5 – 7 – 4 – 3 – 1	24709,2	304290,0	279580,8
	6	1 – 5 – 3 – 7 – 4 – 3 – 1	30804,0	330005,0	299201,0
	7	1 – 5 – 3 – 7 – 3 – 2 – 5 – 1	20658,2	339715,0	319056,8
	8	1 – 5 – 3 – 2 – 7 – 2 – 3 – 5 – 1	16833,3	337525,0	320691,7
12	4	1 – 4 – 2 – 6 – 1	4725,6	246480,0	241754,4
	5	1 – 5 – 2 – 4 – 6 – 1	24153,23	311550,0	287396,8
	6	1 – 5 – 2 – 7 – 4 – 3 – 1	31797,91	343945,0	312147,1
	7	1 – 5 – 3 – 2 – 7 – 4 – 3 – 1	31697,1	356010,0	324312,9
	8	1 – 5 – 3 – 2 – 7 – 4 – 3 – 5 – 1	20223,8	330750,0	310526,2
	9	1 – 5 – 3 – 2 – 7 – 4 – 2 – 3 – 5 – 1	9763,8	339715,0	329961, 2

13	5	1 – 5 – 2 – 4 – 6 – 1	20583,7	311550,0	290966,3
	6	1 – 5 – 2 – 7 – 4 – 6 – 1	30328,29	347780,0	317451,0
	7	1 – 5 – 2 – 3 – 7 – 4 – 3 – 1	31680,9	372985,0	341304,1
	8	1 – 5 – 2 – 3 – 7 – 4 – 3 – 5 – 1	31750,28	384410,0	352659,7
	9	1 – 5 – 3 – 2 – 7 – 4 – 2 – 3 – 5 – 1	17679,4	346150,0	328470,6
14	5	1 – 7 – 2 – 4 – 6 – 1	11113,14	298355,0	287241,9
	6	1 – 5 – 2 – 7 – 4 – 6 – 1	26395,0	347780,0	321385,0
	7	1 – 2 – 5 – 3 – 7 – 4 – 3 – 1	30252,7	376220,0	345967,3
	8	1 – 5 – 2 – 3 – 7 – 4 – 3 – 5 – 1	30578,0	401385,0	370807,0
	9	1 – 5 – 3 – 2 – 7 – 4 – 2 – 3 – 5 – 1	26666,9	400920,0	374253,1
15	5	1 – 7 – 2 – 4 – 6 – 1	11521,0	312785,0	301264,0
	6	1 – 5 – 7 – 4 – 2 – 6 – 1	21426,7	356915,0	335488,3
	7	1 – 2 – 5 – 3 – 7 – 4 – 3 – 1	25857,1	376220,0	350362,9
	8	1 – 5 – 2 – 3 – 7 – 4 – 3 – 5 – 1	26501,1	401385,0	374883,9
	9	1 – 5 – 3 – 2 – 7 – 4 – 2 – 3 – 5 – 1	24103,6	413715,0	389611,4

Table 4. Results for 750 TEU container ship.

Ship speed (knots)	Number of segments (K)	Route	Profit [$]	Revenue [$]	Total costs [$]
11	4	1 – 7 – 2 – 6 – 1	12667,0	332775,0	320108,0
	5	1 – 2 – 7 – 4 – 3 – 1	33178,1	382010,0	348831,9
	6	1 – 5 – 2 – 7 – 3 – 5 – 1	32029,2	435640,0	403610,8
	7	1 – 5 – 3 – 2 – 7 – 3 – 5 – 1	22229,6	417115,0	394885,4
	8	1 – 5 – 3 – 2 – 7 – 2 – 3 – 5 – 1	6231,1	377975,0	371743,9
12	4	1 – 7 – 4 – 6 – 1	12241,2	331615,0	319373,8
	5	1 – 2 – 7 – 4 – 3 – 1	31857,6	401865,0	370007,4
	6	1 – 2 – 7 – 4 – 3 – 5 – 1	36681,3	432825,0	395963,7
	7	1 – 5 – 3 – 7 – 2 – 3 – 5 – 1	28286,4	453710,0	425423,6
	8	1 – 5 – 3 – 2 – 7 – 2 – 3 – 5 – 1	19085,9	444545,0	425459,1
13	4	1 – 4 – 7– 6 – 1	6768,3	331615,0	324846,7
	5	1 – 2 – 7 – 4 – 6 – 1	29170,1	389230,0	360059,9
	6	1 – 5 – 7 – 4 – 3 – 5 – 1	37137,3	448435,0	411297,7
	7	1 – 3 – 2 – 7 – 4 – 3 – 5 – 1	40173,4	463360,0	423186,6
	8	1 – 5 – 3 – 2 – 7 – 4 – 3 – 5 – 1	24690,4	426195,0	401504,6
	9	1 – 5 – 3 – 2 – 3 – 7 – 2 – 3 – 5 – 1	5246,32	430115,0	424868,7
14	5	1 – 2 – 7 – 4 – 6 – 1	24164,7	389230,0	365065,3
	6	1 – 5 – 2 – 7 – 4 – 6 – 1	35545,0	440395,0	404850,0
	7	1 – 3 – 2 – 7 – 4 – 3 – 5 – 1	41218,5	490885,0	449666,5
	8	1 – 5 – 3 – 2 – 7 – 4 – 3 – 5 – 1	35286,2	479740,0	444453,8
	9	1 – 5 – 3 – 2 – 7 – 4 – 7 – 3 – 5 – 1	18282,1	432810,0	414527,9
15	5	1 – 7 – 4 – 3 – 6 – 1	18695,4	405420,0	386724,6
	6	1 – 2 – 7 – 4 – 3 – 6 – 1	38673,1	462770,0	424096,9
	7	1 – 3 – 2 – 7 – 4 – 3 – 5 – 1	36904,1	490885,0	453980,9
	8	1 – 5 – 3 – 2 – 7 – 4 – 3 – 5 – 1	39867,7	526230,0	486362,3
	9	1 – 5 – 3 – 2 – 7 – 4 – 2 – 3 – 5 – 1	27087,4	487860,0	460772,6
16	5	1 – 7 – 2 – 4 – 6 – 1	8189,9	388450,0	380260,1
	6	1 – 2 – 7 – 4 – 3 – 6 – 1	33373,4	462770,0	429396,6
	7	1 – 3 – 2 – 7 – 4 – 3 – 6 – 1	39922,5	494845,0	454922,5
	8	1 – 5 – 3 – 2 – 7 – 4 – 3 – 5 – 1	37961,4	540405,0	502443,6
	9	1 – 5 – 3 – 2 – 7 – 4 – 2 – 3 – 5 – 1	34503,0	532840,0	498337,0
17	6	1 – 5 – 7 – 4 – 3 – 6 – 1	23980,3	468685,0	444704,7
	7	1 – 3 – 2 – 7 – 4 – 3 – 6 – 1	37632,5	508915,0	471282,5
	8	1 – 5 – 3 – 2 – 7 – 4 – 3 – 5 – 1	32973,9	540405,0	507431,1
	9	1 – 5 – 3 – 2 – 7 – 4 – 2 – 3 – 5 – 1	32266,3	553245,0	520978,7

As can be seen from Tables 3, 4 and 5, the shipping company should choose a 750 TEU container ship with sailing speed of 14 knots to operate the route with 7 segments ($K = 7$). The optimal ship route consists of calling at the following ports: Port 1 – Port 3 – Port 2 – Port 7 – Port 4 – Port 3 – Port 5 – Port 1.

Table 5. Results for 1000 TEU container ship.

Ship speed (knots)	Number of segments (*K*)	Route	Profit [$]	Revenue [$]	Total costs [$]
13	5	1 – 2 – 7 – 2 – 6 – 1	20488,0	480110,0	459622,0
	6	1 – 5 – 2 – 7 – 4 – 3 – 1	30616,6	516100,0	485483,4
	7	1 – 5 – 2 – 7 – 4 – 3 – 5 – 1	15976,8	485385,0	469408,2
14	5	1 – 2 – 7 – 4 – 6 – 1	17004,2	466230,0	449225,8
	6	1 – 5 – 2 – 7 – 4 – 3 – 1	32866,0	540265,0	507399,0
	7	1 – 5 – 2 – 7 – 4 – 3 – 5 – 1	28312,7	538775,0	510462,3
	8	1 – 5 – 3 – 2 – 7 – 4 – 3 – 5 – 1	13000,5	510090,0	497089,5
15	5	1 – 7 – 4 – 3 – 6 – 1	14425,8	482345,0	467919,2
	6	1 – 5 – 2 – 7 – 4 – 6 – 1	28137,1	522785,0	494647,9
	7	1 – 5 – 2 – 7 – 4 – 3 – 5 – 1	34268,1	580515,0	546246,9
	8	1 – 5 – 3 – 2 – 7 – 4 – 3 – 5 – 1	21937,8	555480,0	533542,2
	9	1 – 5 – 3 – 2 – 7 – 4 – 2 – 3 – 5 – 1	1308,9	512865,0	511556,1
16	5	1 – 7 – 4 – 3 – 6 – 1	8473,4	482345,0	473871,6
	6	1 – 2 – 7 – 4 – 3 – 6 – 1	29223,7	540515,0	511291,3
	7	1 – 5 – 2 – 7 – 4 – 3 – 5 – 1	30903,3	590465,0	559561,7
	8	1 – 5 – 3 – 2 – 7 – 4 – 3 – 5 – 1	26960,5	598695,0	571734,5
	9	1 – 5 – 3 – 2 – 7 – 4 – 2 – 3 – 5 – 1	8695,9	556035,0	547339,1
17	6	1 – 2 – 7 – 4 – 3 – 6 – 1	22777,2	540515,0	517737,8
	7	1 – 5 – 2 – 7 – 4 – 3 – 5 – 1	25314,5	590465,0	565150,5
	8	1 – 5 – 3 – 2 – 7 – 4 – 3 – 5 – 1	27114,8	630210,0	603095,2
	9	1 – 5 – 3 – 2 – 7 – 4 – 2 – 3 – 5 – 1	13087,3	595175,0	582087,7
18	6	1 – 2 – 7 – 4 – 3 – 6 – 1	15939,9	540515,0	524575,1
	7	1 – 5 – 2 – 7 – 4 – 3 – 6 – 1	26736,7	598665,0	571928,3
	8	1 – 5 – 3 – 2 – 7 – 4 – 3 – 5 – 1	23849,2	644385,0	620535,8
	9	1 – 5 – 3 – 2 – 7 – 4 – 2 – 3 – 5 – 1	14945,3	628700,0	613754,7
19	6	1 – 5 – 7 – 4 – 3 – 6 – 1	3590,8	553390,0	549799,2
	7	1 – 5 – 2 – 7 – 4 – 3 – 6 – 1	24943,2	621375,0	596431,8
	8	1 – 5 – 3 – 2 – 7 – 5 – 3 – 5 – 1	17459,5	644385,0	626925,5
	9	1 – 5 – 3 – 2 – 7 – 4 – 2 – 3 – 5 – 1	13524,6	646770,0	633245,4

6. Conclusion

As the volume of containers transported worldwide continuously increases, the organisation of container transport services becomes more and more complex and the need to improve the performance of services is gaining importance. This paper deals with one aspect of the organisation of container transportation and this concern the routing of feeder container ships. We addressed the feeder container ship routing problem as a problem of maximizing the shipping company profit while picking up and delivering containers with empty container repositioning.

We propose a mixed integer linear programming (MILP) formulation for the considered problem and show that it can simplify the decision making process in the shipping company engaged in the feeder service. In particular, we resolve routing decisions and sizing of container batches in calling ports for feeder container ships. Our results indicate that the planning process in the container shipping company could be improved by applying the proposed decision support system based on optimization routine, which would significantly impact the business results of a shipping company. The model presented in this paper could be a very useful practical tool for container carriers to make long term strategic decisions about establishing feeder container transport services. They can solve their practical problems, test different solutions of the problems and choose those which are most suitable for their own needs. Therefore, the proposed method provides a quick insight into potential business results, the profitability of the feeder container ships in particular. In addition, it considerably facilitates the decision making process in a shipping company engaged in feeder container service.

We treated this problem by applying CPLEX 11.2 MIP solver and AMPL for Linux operating system. This solver was able to optimally solve small size problem instances with maximum seven calling ports. However, like in many other combinatorial optimization problems, real-life situations may be too complex to be solved to optimality within a reasonable amount of time. Therefore, the use of metaheuristic approaches which are the common way to tackle these kinds of problems is obviously a further research direction.

Moreover, this study can be extended in several directions. The extension to a multi-ship problem is straightforward. Stochastic modelling of some parameters (like container demands, service time) may also be included, although it will significantly increase the complexity of the problem. In addition, the lack of real data for the considered problem imposed us to generate random test examples. Therefore, verification of the proposed formulation through some real situations could be of particular importance for the future research.

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