



# Theoretical Analysis of the Relationship between Accident Rate of a Fleet of Vessels and Age Distribution of the Fleet

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ABSTRACT

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It is well accepted that the age of a seagoing vessel significantly influences her probability of being involved in a marine accident. This implies that the accident rate of a fleet of vessels (e.g. the world fleet as a whole, individual shipping company fleet, etc.) will depend on the age structure of the fleet. Perhaps due to its seeming obviousness, this fact has hitherto (to the author’s knowledge) not been given careful consideration in the maritime safety literature. This paper attempts to fill this gap by developing a theoretical equation that relates accident rate of a fleet of vessels to numerical characteristics of its age distribution. The equation shows that the fleet accident rate depends not only on the average age of ships, but also on other parameters of the age distribution of the fleet.

## 1. The Analysis

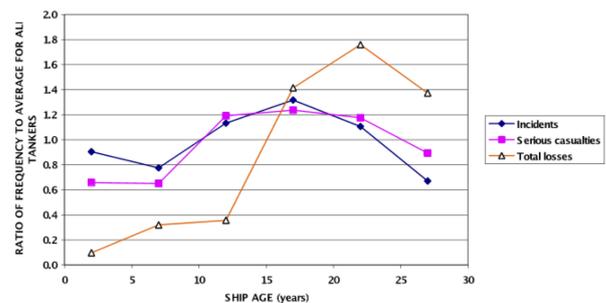
It is a well accepted fact that the age of a seagoing vessel significantly influences her probability of being involved in a marine accident. According to (DNV, 2001), the frequency of incidents and serious casualties is twice as high for tankers aged 15-20 years compared with ships aged 0-5 years (see (Figure 1)). It is obvious that because of this fact, the accident rate of a fleet of vessels (e.g. the world fleet as a whole, individual shipping company fleet, etc.) will have a significant dependence on the age structure of the fleet. Perhaps due to its seeming obviousness, this fact has hitherto (to the author’s knowledge) not been given careful consideration in the maritime safety literature. To fill this gap let us develop an equation that relates accident rate of a fleet to numerical characteristics of its age distribution.

We define the accident rate of a fleet of vessels,  $K$ , as the ratio

$$K = \frac{NA}{N}$$

where  $NA$  is the expected number of accidents occurring during a certain time period, and  $N$  is the total number of ships

Figure 1: The dependence of the tanker accident rate on vessel age.



Source: OGP (2010) Water Transport Accident Statistics. Report No. 434 - 10

in the fleet. Similarly, the accident rate for ships of  $i$ th age,  $C_i$ , is defined as

$$C_i = \frac{NA_i}{N_i}$$

Where  $NA_i$  is the expected number of accidents with ships of  $i$ th age and  $N_i$  is the total number of ships of  $i$ th age in the fleet. Obviously, the following equation holds:

$$K = \sum_i \left( \frac{NA_i}{N_i} \cdot \frac{N_i}{N} \right) = \sum_i C_i B_i \quad (1)$$

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Where  $B_i = N_i / N$  is the proportion of ships of  $i$ th age within the fleet. According to formula (4), the accident rate dependence on the fleet age structure is determined by the value of  $C_i$ . It is obvious that  $C_i$  is a random variable. Therefore, it does not make sense to discuss any sort of function that could uniquely determine the value of  $C_i$ . We can only speak of the mathematical expectation of the random variable  $C_i$  - which we will denote as  $c(i)$  - which is a certain function of vessel age. As can be seen in (Figure 1), this function, obviously, must have a non-linear character. From a mathematical standpoint, this means that the function  $c(i)$  must have higher-order derivatives. Let us assume that  $c(i)$  is a certain analytic function, which can be represented as a Taylor series expansion, in positive integral powers of  $(i - a)$ . Setting  $a = i_1$ , where  $i_1$  is the average age of vessels in the fleet, and disregarding any terms containing derivatives higher than the fourth degree for the sake of simplicity, we write the expansion  $c(i)$  in the form

$$c(i) = c(i_1) + c'(i_1)(i - i_1) + \frac{c''(i_1)}{2!}(i - i_1)^2 + \frac{c'''(i_1)}{3!}(i - i_1)^3 + \frac{c^{(4)}(i_1)}{4!}(i - i_1)^4$$

After substituting the last expression into (3), we will obtain

$$E(K) = c(i_1) \sum B_i + c'(i_1) \sum B_i(i - i_1) + \frac{c''(i_1)}{2!} \sum B_i(i - i_1)^2 + \frac{c'''(i_1)}{3!} \sum B_i(i - i_1)^3 + \frac{c^{(4)}(i_1)}{4!} \sum B_i(i - i_1)^4 \quad (2)$$

Where  $E(K)$  is the mathematical expectation of random variable  $K$ . Noting further, that the sum of the form

$$\sum_1 B_i(i - i_1)^n$$

Is the  $n$ -th central moment of the fleet age distribution, and taking into account that

$$\sum_1 B_i = 1$$

We can rewrite expression (5) in the form

$$E(K) = c(i_1) + c'(i_1)\mu_1 + \frac{c''(i_1)}{2}\mu_2 + \frac{c'''(i_1)}{6}\mu_3 + \frac{c^{(4)}(i_1)}{24}\mu_4$$

Where  $\mu_1, \mu_2, \mu_3$  and  $\mu_4$  are the central moments of vessel age distribution of the 1st, 2nd, 3rd, and the 4th order, respectively. Taking into account that  $\mu_1 = 0$ , and using following known relationships for the remaining moments:

$$\mu_2 = \sigma^2, \mu_3 = \gamma_1\sigma^3, \mu_4 = (\gamma_2 + 3)\sigma^4$$

Where  $\sigma, \gamma_1$  and  $\gamma_2$  are respectively, the standard deviation, skewness, and kurtosis of the vessel age distribution, we will finally get:

$$E(K) = c(i_1) + \frac{c''(i_1)}{2}\sigma^2 + \frac{c'''(i_1)}{6}\gamma_1\sigma^3 + \frac{c^{(4)}(i_1)}{24}(\gamma_2 + 3)\sigma^4 \quad (3)$$

Thus, we arrive at the conclusion that the fleet accident rate depends not only on the average age  $i_1$  of the fleet's vessels, but also on other numerical age structure characteristics, particularly the standard deviation,  $\sigma$  of the vessel age. It should be noted that this result was obtained using the nonlinear behaviour of the vessel accident rate dependence on age, and therefore is not connected to any specific form of the function  $c(i)$  (provided that the latter belongs to the class of analytic functions).

## 2. Conclusions

The obtained conclusion is a hypothesis. We could test the hypothesis by comparing the statistical data relating to the accident rate of the world fleet to age structure data over a sufficiently long period of time, using mathematical statistics methods. However, in trying to do this, we come upon one major difficulty. This difficulty lies in the fact that the published statistics on the age structure of the world fleet are actually limited to only one parameter - the average age of vessels. Other parameters of the fleet age structure, which, according to equation 3, can also have an effect on the level of the accident rate of the fleet, are not subject to systematic calculation. In addition, the study is complicated by the fact that laid-up ships, or those that are not in operation for other reasons, do not contribute to the accident rate of the fleet. Therefore, we need to know the age composition of only the active fleet, versus the entire fleet as a whole. However, such data is also unpublished. The author hopes that this article will serve as the basis for the maritime community to start performing systematic statistical analysis of not only the average age of the vessels, but also of other age structure parameters that are part of equation 3, both for the world fleet and the active and (or) inactive fleets.

## References

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