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A Funnel Control based Terminal Sliding Mode Control using a robust observer for Autonomous Underwater Vehicles in the vertical Plane

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ABSTRACT

This article is devoted to the advanced control of a vertically autonomous underwater robots that are used for the assignment of various search and rescue operations on the high seas. The kinematic and dynamic models of the robot are described by the differential equations with six degrees of freedom with respect to the Earth frame. Due to the uncertain marine environment, these equations are nonlinear and strongly coupled. This control is a combination of funnel-based control and sliding mode for the velocity and depth control; which consists in taking the funnel control as much as a time-varying coefficient of the sliding surface. However, the implementation of this control faces difficulties caused by non-measurable variables, which are resolved by the addition of an extended super-twisting observer. The stability study using the Lyponuv method and simulation results confirm the effectiveness of the proposed method for controlling the speed and trajectory of AUVs in the vertical plane.

1. Introduction.

Human beings still want to reduce their efforts for the exploitation of the seabed, maintenance of dams and pipeline installations; using submarine robots [1] which are classified according to their decisional and energy autonomy [2, 3]. For a great freedom of movement, the solution is to use an autonomous robot that can transmit its location to the satellite navigation and tracking system [4-6]. There are a number of scientific challenges that AUVs can face; knowing that its dynamic model is non-linear and highly coupled. The vehicle is a nonholonomic system that depends on an uncertain path due to the currents influencing its model [7]. The hydrodynamic coefficients are often poorly known, since their measurement with physical sensors and their estimation is almost impossible in the presence of disturbances, which have consequences on the trajectory tracking [8, 9].

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However, several research projects are being conducted to address the aforementioned difficulties. In the research done in [10] the authors have developed a disturbance observer for the purpose of estimating unknown and unstable environmental disturbances over time, besides that a dynamic controller distributed to have the parameters of position and speed. [11] proposed a third-order continuous FFTESO in order to estimate the group disturbances and their first derivatives of a fully actuated AUV. By comparing this approach with GESO and FTESO, they proved that the convergence of the system is maintained, faster and more accurate in a finite time, which will greatly depend on the choice of the initial state of the system. The simulation results in [12] show that the combination of an ESO and a finite-time control solves the chattering problem and the uncertainty of the AUV model, by continuously observing on convergence speed in a finite time and robustness. The authors in [13] adapted the funnel constraints to dynamically adjust the robot trajectory in a complex environment, while maintaining the desired performance. J. Andrich et al. [14] studied the control of an Autonomous Underwater Vehicle (AUV) by using (PFM) in order to allow a visual guiding of AUV. The aim is to track pipelines and cables. The proposed method is assessed and compared to other known researches within the same topic.

Kotov et al. [15] explored the potential use of autonomous underwater vehicles for mineral resource exploration and extraction in the Russian Federation and its continental shelf. Additionally, they conducted a comprehensive review of the current status, applications, and limitations of the global utilization of these vehicles.

To improve quadrotors robustness, [16] proposed an unknown input observers (UIOs) for trajectory and attitude with asymptotic error convergence. Next, an appointed-time funnel control for the regularization of the trajectory tracking error, avoiding the singularity problem.

The main purpose of this paper is to control an AUV in diving plane, by combining a funnel-based control and a sliding mode control using an extended super-twisting observer for the estimation of non-measurable parameters; where the forward speed and the vehicle's depth represent the output variables. A relative degree of nature two is attributed to the depth subsystems. The idea of this command is to consider the funnel-based control as much as a time-varying coefficient of the sliding surface, and also to design a control of the vehicle's forward velocity, unlike previous studies where they consider it to be constant. This approach incorporates an additional control term thus improving the performance of the proposed controller.

This paper is structured as follows. The second section provides the AUV's mathematical representation in the vertical plane. The third section, explains the structure of the observer and its stability. In the fourth section, the velocity control and depth control are proposed. In the fifth section, the efficiency results of the proposed method are demonstrated by means of computer simulations. The final section concludes the paper.

2. Mathematical modeling of AUV.

The general motion dynamics of the AUV are designed by the kinematic and dynamic motion of this rigid body with 6 degrees of freedom (DOF) [17, 18]. This dynamic will be represented by 12 first-order differential equations, coupled and with constant coefficients [19]. In order to create control laws that are simpler, we are interested in the diving plane whose control surfaces are horizontally zero. The following equations (1) represent the heave and pitch equations of motion of the vehicle in the body-fixed coordinate frame:

$$m(\dot{u} + q\omega) = X_{qq}q^2 + X_{\dot{u}}\dot{u} + X_{\omega q}\omega q + X_{q\delta}uq\delta$$
$$+X_{\omega\omega}\omega^2 + X_{\omega\delta}\delta u\omega + X_{\delta\delta}u^2\delta^2$$
$$-(W-B)\sin\theta + F_P$$
$$m(\dot{\omega} - uq) = Z_{\dot{q}}\dot{q} + Z_{\dot{\omega}}\omega + Z_{uq}uqZ_{u\omega}u\omega + Z_{uu}u^2\delta$$
$$+(W-B)\cos\theta + Z_{\omega|\omega|}\omega|\omega|$$
$$+Z_{q|q|}q|q| + Z_H$$
$$I_{yy}\dot{q} = M_{\dot{q}}\dot{q} + M_{\dot{\omega}}\dot{\omega} + M_{uq}uq + M_{u\omega}u\omega + M_{uu}u^2\delta$$
$$-(z_G W - z_B B)\sin\theta$$
$$-(x_G W - x_B B)\cos\theta + M_{\omega|\omega|}\omega|\omega|$$
$$+M_{q|q|}q|q| + M_P$$
$$\dot{\theta} = q$$
$$\dot{z} = -u\sin\theta + w\cos\theta \tag{1}$$

Where; u is the vehicle's forward velocity, ω is the heave velocity, θ is the pitch angle (θ≠π/2+kπ), q is the pitch angle velocity, z is the vehicle's depth, δ is the control fin angle, Fp is the propulsion force which control the forward velocity, m is the mass of the vehicle, I_yy is the moment of inertia of the vehicle about the pitch axis, W denotes the vehicle's weight and B is the vehicle buoyancy, Z_q·, Z_uq, Z_ω·, M_q· … are the hydrodynamics parameters. Lastly, M_P and Z_H identify the cross- flow drag terms and evaluate them as disturbances. The state vector is as follows:

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix}^T = \begin{bmatrix} u & \omega & q & \theta & z \end{bmatrix}^T \tag{2}$$

The control vector:

$$v = \begin{bmatrix} F_P & \delta \end{bmatrix}^T \tag{3}$$

And the inertial matrix:

$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 & 0 & 0 \\ 0 & m - Z_{\dot{\omega}} & -Z_{\dot{q}} & 0 & 0 \\ 0 & -M_{\dot{\omega}} & I_{yy} - M_{\dot{q}} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

In state space forms, the vehicle dynamics (1) are represented as:

$$M\dot{x} = f_x(x,t) + g_x(x,v) \tag{5}$$

Where

$$\dot{x} = f(x,t) + g(x,v) \tag{6}$$

Whose outputs are:

$$y_1 = u\,,\quad y_2 = z \tag{7}$$

With

$$f(x,t) = M^{-1}f_x(x,t) \tag{8}$$

And

$$g(x,v) = M^{-1}g_x(x,v) \tag{9}$$

3. Nonlinear observer design.

Assume that just the system output u and z are measured, it needs to estimate the other state variables w, q and θ.

Define the new variables:

$$\xi = [\xi_1, \xi_2, \xi_3]^T = [z,\ u\sin\theta\,, w\cos\theta\,]^T \tag{10}$$

Then;

$$\dot{\xi}_1 = -\xi_2 + \xi_3$$

$$\dot{\xi}_2 = uq\cos\theta + \dot{u}\sin\theta \quad (11)$$

$$\dot{\xi}_2 = -wq\sin\theta + \cos\theta\,(f_w(x,t) - g_w(x)\delta)$$

The state observer is chosen as:

$$\dot{\widehat{\xi}}_1 = -\widehat{\xi}_2 + \widehat{\xi}_3 + \Gamma_1$$

$$\dot{\widehat{\xi}}_2 = u\hat{q}\cos\widehat{\theta} + \Gamma_2 \quad (12)$$

$$\dot{\widehat{\xi}}_2 = u\hat{q}\cos\widehat{\theta} + \Gamma_2\#(12)$$

$$\dot{\widehat{\xi}}_3 = -\hat{w}\hat{q}\sin\widehat{\theta} + \cos\widehat{\theta}\,(f_w(\hat{x}) - g_w(\hat{x})\,\delta) + \Gamma_3$$

Where:

$$\Gamma_1 = \mu_1\left|\widetilde{\xi}_1\right|^{2/3} sign(\widetilde{\xi}_1)$$

$$\Gamma_2 = \mu_2\left|\widetilde{\xi}_1\right|^{1/3} sign\left(\widetilde{\xi}_1\right) \quad (13)$$

$$\Gamma_3 = \mu_3 sign(\widetilde{\xi}_1)$$

And μ_1, μ_2, μ_3 are gains to be chosen according to (LEVANT, 1998) and (LEVANT, 2003) where:

$$\widetilde{\xi}_1 = \xi_1 - \widehat{\xi}_1$$

$$\widetilde{\xi}_2 = \xi_2 - \widehat{\xi}_2 \quad (14)$$

$$\widetilde{\xi}_3 = \xi_3 - \widehat{\xi}_3$$

One choice of parameters that meets the requirements in (LEVENT, 1998) and (LEVANT, 2003) is according to (CHALANGA ET AL, 2016), $\mu_1 = 6L^{\frac{1}{3}}$, $\mu_2 = 11L^{\frac{1}{2}}$ and $\mu_3 = 6L$, where L is a sufficiently large constant.

The error dynamics of the state observer can be written as:

$$\dot{\widetilde{\xi}}_1 = -\widetilde{\xi}_2 + \widetilde{\xi}_3 - \mu_1\left|\widetilde{\xi}_1\right|^{2/3} sign(\widetilde{\xi}_1)$$

$$\dot{\widetilde{\xi}}_2 = \tilde{F}_1 - \mu_2\left|\widetilde{\xi}_1\right|^{1/3} sign\left(\widetilde{\xi}_1\right) \quad (15)$$

$$\dot{\widetilde{\xi}}_3 = \tilde{F}_2 - k_3 sign(\widetilde{\xi}_1)$$

Where

$$\tilde{F}_1 = u\left(q\cos\theta - \hat{q}\cos\widehat{\theta}\right) \quad (16)$$

$$\tilde{F}_2 =$$

$$-wq\sin\theta + \cos\theta\,(f_w(x,t) - g_w(x)\,\delta)$$

$$+\hat{w}\hat{q}\sin\widehat{\theta} - \cos\widehat{\theta}\,(f_w(\hat{x}) - g_w(\hat{x})\,\delta) \quad (17)$$

If $\left|\tilde{F}_1\right| < \Delta_1$ and $\left|\tilde{F}_2\right| < \Delta_2$

Then the state observer errors goes to zero in finite time (MORENO, 2012). Since the AUV is a mechanical system then $\tilde{F}_1$ and $\tilde{F}_2$ will not change infinitely fast. It is therefore a valid assumption to assume that $\tilde{F}_1$ and $\tilde{F}_2$ are bounded.

4. Controller design.

This section provides a list of necessary definitions and theorems.

Definition 1: The performance funnel is based on a bounded function $\psi(t)$ where:

$$\psi(t) = \frac{1}{\varphi(t)} \quad (18)$$

And

$$F_\varphi := \{(t, e) \in R_{\geq 0} \times R \parallel e(t) < \psi(t)\} \quad (19)$$

The purpose of funnel-based control is to create a shape for the tracking error's transient response and place it in the funnel.

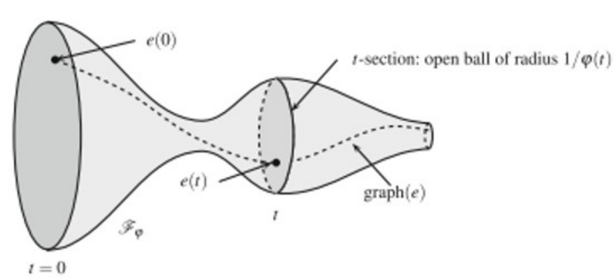
The funnel structure F_φ and the path of the output tracking error e(t) are depicted in Figure 1. The funnel's shape is effective in achieving transient response characteristics such as overshoot, rise time, time settling and so forth in this approach.

Make $\varphi(t)$ a component of the class that follows:

$$\Phi := \left\{ \begin{array}{c} \varphi(t) \in w^{1,\infty}(R_+, R_+)| \\ \forall t > 0 : \varphi(t) > 0 \end{array} \right\} \quad (20)$$

And

Figure 1: Performance funnel.



Source: Authors.

$$\lim_{\tau\to\infty} \inf \varphi(t) \ > 0, \forall\tau > 0 : \ \varphi^{-1}(.) \quad (21)$$

is globally Lipschitz.

$w^{1,\infty}(R_+, R_+)$ is a classification of function that have limited derivatives. The funnel's boundaries $\psi(t)$ are influenced by the appropriate function selection of $\varphi(t)$, with the objective is to keep the error within the funnel F_φ. Considering that:

$$\lambda_1 = sup_{t\epsilon(0,\infty)}\psi(t), \phi^* := inf_{t\epsilon(0,\infty)}\psi(t) \quad (22)$$

Also, $\varphi(t) \geq \frac{1}{\lambda_1}; \forall t \geq T$ where T are relatively large, in the end e(t) is confined by λ_1. In Ilchmann et al. (2002), a funnel-based control with the structure

$$u(t) = -k(t)\,e(t) \quad (23)$$

was utilized to address the issue of output tracking for a dynamical system with relative degree one. $k(t)$ is a time-varying gain that is based on the funnel shape and was chosen as the solution:

$$k(t) = \frac{\varphi(t)}{1-\varphi(t)|e(t)|} = \frac{1}{\psi(t)-|e(t)|} \quad (24)$$

This controller prevents errors from approaching the funnel boundary by increasing the gain when they approach it, unlike the adaptive approach which is dynamically generated.

4.1. Velocity control.

The objective in this part is to control the forward speed, for which you need to choose the most suitable F_p control.

Take $x_1 = y_1 = u$ as an output variable, and u_d as a constant reference.

A sliding manifold is referred to as such because x_1 has relative degree one:

$$\sigma = x_1 - u_d \tag{25}$$

By deriving it in terms of time:

$$\dot{\sigma} = f_u(x,t,\delta) + g_u F_p \tag{26}$$

Knowing that F_p is assumed to be a combination of funnel based-control and sliding mode as follows:

$$F_P = v_{eq} + v_1 + v_2 \tag{27}$$

With:

$$v_{eq} = \frac{-k_0 z_1 - \varepsilon k_1 f_u(x,t,\delta) + \varepsilon k_1 \dot{u}_d}{\varepsilon k_1 g_u} \tag{28}$$

$$v_1 = -\beta sign(\sigma) \tag{29}$$

$$v_2 = -\frac{1}{k_1 g_u} k(t)\,\sigma(t) \tag{30}$$

Where the constants $k_0,\ k_1, \varepsilon > 0$, ε infinitely small and the time-varying gain $k(t)$ of the funnel depends on its shape and sliding surface:

$$k(t) = \frac{1}{\psi(t) - |\sigma(t)|} \tag{31}$$

As a result, the closed-loops system that follows:

$$\dot{\sigma} = \frac{-k_0 z_1 + \varepsilon k_1 \dot{u}_d}{\varepsilon k_1} - \frac{1}{k_1} k(t)\,\sigma(t) - g_u \beta sign(\sigma) \tag{32}$$

Considering the sliding mode manifold S0 is designed as:

$$S_0 = \frac{\sigma(t)}{F_\varphi(t) - |\sigma(t)|} \tag{33}$$

With:

$$\sigma = x_1 - u_d \tag{34}$$

$$F_\varphi(t) = \gamma_0\, e^{-a_0 t} + \gamma_\infty \tag{35}$$

When:

$$\gamma_0 \geq \gamma_\infty > 0\,;\, a_0 > 0\,; \qquad \gamma_\infty = \lim_{t \to +\infty} F_\varphi(t)$$

The derivative of (1) can be calculated as:

$$\dot{S}_0 = F_\varphi\, \Phi_F \dot{\gamma} - \dot{F}_\varphi \Phi_F\, \gamma \tag{36}$$

$$\Phi_F = \frac{1}{\left(F_\varphi(t) - |\sigma(t)|\right)^2} \tag{37}$$

$$\dot{S}_0 = F_\varphi\, \Phi_F \left[f_u(x,t,\delta) + g_u F_p\right] - \dot{F}_\varphi \Phi_F\, \gamma \tag{38}$$

To make S_0 converge to zero within a finite time the non-singular terminal sliding mode manifold is designed as:

$$\beta \left|\dot{S}_0\right|^{q/p} sign(S_0) + S_0 = 0 \tag{39}$$

Where $\beta > 0$; p and q are positive add integers with $p < q$
Substituting (36) into (39), the controller is designed as:

$$F_p = \frac{1}{g_u} \left[\frac{\dot{F}_\varphi}{F_\varphi}\gamma - \frac{1}{F_\varphi\, \Phi_F} \frac{1}{\beta} |S_0|^{q/p}\, sign(S_0) - \dot{u}_d - f_u(x,t,\delta)\right] \tag{40}$$

Stability analysis:

Lemma 1: Assuming there is a $V(t)$ continuous positive definite function, which satisfies the following inequality:

$$\dot{V}(t) + nV^\gamma(t) \leq 0 \tag{41}$$

$$(\forall t > t_0\ ,\ the\ constant\ n > 0$$

$$\forall\, t_0 :$$

$$V^{1-\gamma}(t) \leq V^{1-\gamma}(t_0) - n(1-\gamma)(t - t_0)$$

$$\begin{pmatrix} t_0 \leq t \leq t_s,\ V(t) \equiv 0\ (\forall t \geq\ t_s)\ and \\ t_s \leq t_0 + \frac{V^{1-\gamma}(t_0)}{n(1-\gamma)} \end{pmatrix} \tag{42}$$

Theorem:
Consider the sub-system (1) with the control law (40) , then the tracking error σ is bounded.

Proof:
Choose the following Lyapunov function:

$$V_1 = \frac{1}{2} {S_0}^2 \tag{43}$$

Differentiating (43), we have:

$$\dot{V}_1 = S_0 \dot{S}_0 \tag{44}$$

$$\dot{V}_1 = S_0 \left[F_\varphi\, \Phi_F \left[f_u(x,t,\delta) + g_u F_p\right] - \dot{F}_\varphi \Phi_F\, \gamma\right] \tag{45}$$

Substituting (40) into (45) yields:

$$\dot{V}_1 \leq -\frac{1}{\beta} |S_0|^{(p+q)/q} \leq 0 \tag{46}$$

Inequality (46) implies that S_0 is bounded then the stability of the sub-system (1) with the control law (40)

Now, we need to further prove the finite time convergence. We have:

$$\dot{V}_1 \leq -\frac{1}{\beta} |S_0|^{(p+q)/q}$$

$$\leq -\frac{1}{\beta} (2)^{(p+q)/(2q)} \left(\frac{1}{2} {S_0}^2\right)^{(p+q)/(2q)} \tag{47}$$

$$\leq\ -\frac{1}{\beta}\,(2)^{(p+q)/(2q)}\ V_1{}^{(p+q)/(2q)} \tag{48}$$

$$\leq\ -k_1\ V_1{}^{k_2} \tag{49}$$

Where

$$k_1 =\ \frac{1}{\beta}2^{(p+q)/(2q)} \tag{50}$$

$$k_2 =\ (p+q)/(2q) \tag{51}$$

These, we can obtain:

$$\dot{V}_1 +\ k_1\ V_1{}^{k_2}\ \leq 0 \tag{52}$$

According to lemma (1), it can be calculated that the terminal sliding manifold so can converge to the equilibrium point within a finite time t_1 given by:

$$t_1 = \frac{V^{1-K_2(t_0)}}{k_1(1-\ K_2)} \tag{53}$$

4.2. Depth control.

The objective in this part is to regulate depth by acquiring a constant reference for the variable z.

Take $\overline{x}_2 = y_2 = z$ as an output variable and considering the new variable $\dot{\overline{x}}_2 = \overline{x}_3 = -u\sin\theta\ + \omega\cos\theta$, the depth subsystem is:

$$\dot{\overline{x}}_2 = \overline{x}_3$$

$$\dot{\overline{x}}_3 = \overline{f}_3\,(x,t) - \overline{g}_3\,(x)\,\delta \tag{54}$$

With:

$$\overline{f}_3\,(x,t) = -u\mathrm{q}\cos\theta\ - \omega q\sin\theta\ + \cos\theta\, f_\omega\,(x,t)$$

$$\overline{g}_3\,(x) = \cos\theta\, g_\omega\,(x) \tag{55}$$

The sliding manifold s_2 is represented as follows:

$$s_2 = \dot{\sigma}_2 + \alpha_2\sigma_2 \tag{56}$$

$$(\alpha_2 > 0)$$

Where:

$$\sigma_2 = \frac{e_z}{F_{\varphi_z} - |e_z|} \tag{57}$$

By deriving it in terms of time:

$$\dot{\sigma}_2 = F_{\varphi_2}\varphi_{F_2}\dot{e}_z - \dot{F}_{\varphi_2}\varphi_{F_2}e_z \tag{58}$$

With:

$$\varphi_{F_2} = \frac{1}{(F_{\varphi_2} - |e_z|)^2} \tag{59}$$

Deriving σ_2 for a second time:

$$\ddot{\sigma}_2 = F_{\varphi_2}\varphi_{F_2}\ddot{e}_z + \Gamma\,(x,e,t) \tag{60}$$

Where:

$$\Gamma\,(x,e_z,t) = F_{\varphi_2}\dot{\varphi}_{F_2}\dot{e}_z + \dot{F}_{\varphi_2}\varphi_{F_2}\dot{e}_z - \dot{F}_{\varphi_2}\dot{\varphi}_{F_2}e_z - \dot{F}_{\varphi_2}\varphi_{F_2}\dot{e}_z - \ddot{F}_{\varphi_2}\varphi_{F_2}e_z \tag{61}$$

The derivative of s_2 is:

$$\dot{s}_2 = \ddot{\sigma}_2 + \alpha_2\dot{\sigma}_2 \tag{62}$$

$$\dot{s}_2 = F_{\varphi_2}\varphi_{F_2}\left(f_3\,(\overline{x},t) - g_3\,(\overline{x})\,\delta - \dddot{z}^d\right) + \Gamma\,(x,e_z,t) + \alpha_2\dot{\sigma}_2 \tag{63}$$

$$\dot{s}_2 = F_{\varphi_2}\varphi_{F_2}\left(\Sigma\,(\overline{x},e_z,t) - g_3\,(\overline{x})\,\delta - \dddot{z}^d\right) + \alpha_2\dot{\sigma}_2 \tag{64}$$

The nonlinear function $\Sigma(\overline{x},t)$ is:

$$\Sigma\,(\overline{x},e_z,t) = f_3\,(\overline{x},t) + \frac{\Gamma(\overline{x},e_z,t)}{F_{\varphi_2}\varphi_{F_2}} \tag{65}$$

To ensure that s_2 converge to zero in a finite time, the sliding mode collector is designed as follows:

$$s_2 + \beta_2|\dot{s}_2|^{q_2/p_2}\, sign\,(s_2) = 0 \tag{66}$$

$$(\beta_2, q_2, p_2 0\ and\ p < q\ )$$

The control law is designed as:

$$\delta = -\frac{1}{g_3(\overline{x})}\left[-\ddot{z}_d + \sum(\overline{x},e_z,t) + \frac{1}{F_\varphi\varphi_{F_2}}\left(\alpha_2\dot{\sigma}_2 + \frac{1}{\beta_2}|S_2|^{q_2/p_2}\, sign\,(S_2)\right)\right] \tag{67}$$

Theorem 2:

For the subsystem expressed by (54) and using the control in (67), the vehicle depth will converge to their reference in finite time.

Proof:

By choosing the Lypunov function as follows:

$$V_2 = \frac{1}{2}s_2{}^2 \tag{68}$$

By deriving it in terms of time:

$$\dot{V}_2 = s_2\dot{s}_2 \tag{69}$$

By replacing $\dot{s}_2$:

$$\dot{V}_2 = S_2\left[F_{\varphi_2}\varphi_{F_2}\,(\sum(\overline{x},e_z,t) - g_3\,(\overline{x})\,\delta - \ddot{z}_d) + \alpha_2\dot{\sigma}_2\right] \tag{70}$$

Substituting (67) into (70) yield:

$$\dot{V}_2 \leq -\frac{1}{\beta_2}|S_2|^{(p_2+q_2)/q_2} \leq 0 \tag{71}$$

$$\dot{V}_2 \leq -\frac{1}{\beta_2}2^{(p_2+q_2)/(2q_2)}V_2{}^{(p_2+q_2)/(2q_2)} \tag{72}$$

$$\dot{V}_2 \leq -K_3V_2{}^{K_4} \tag{73}$$

Where

$$K_3 = \frac{1}{\beta_2}2^{(p_2+q_2)/(2q_2)} \tag{74}$$

And

$$K_4 = (p_2+q_2)/(2q_2) \tag{75}$$

Then, we can obtain:

$$\dot{V}_2 + K_3V_2{}^{K_4} \leq 0 \tag{76}$$

Hence, it can be concluded that the sliding manifold S_2 can converge to the equilibrium point within a finite time t_2 given by:

$$t_2 = \frac{V_2{}^{(1-K_4)}(t_0)}{K_3(1-K_4)} \tag{77}$$

5. Simulation Results.

Numerical simulations for the closed-loop system were performed in order to show the effectiveness of the proposed scheme.

The vehicle model parameters are shown in Table

1. The forward speed u reference is set to 2 m/s and the depth z reference is set to 20 m.

Table 1: AUV model parameters and controller parameters design.

Parameter	Value
W	$299\,N$
m	$30.48\,kg$
I_{yy}	$3.45\,kg.m^2$
z_G	0
z_B	$-0.0196\,m$
x_G	0
y_G	0
x_B	0
y_B	0
B	$306\,N$
X_{qq}	-1.5
$X_{\dot{u}}$	-16
$X_{\omega q}$	-20
$X_{q\delta}$	2.5
$X_{\omega\omega}$	17
$X_{\omega\delta}$	4.6
$X_{\delta\delta}$	1
$Z_{\dot{q}}$	$-1.93\,kg.m/rad$
$Z_{\dot{\omega}}$	$-35.5\,kg$
Z_{uq}	$-5.22\,kg/rad$
$Z_{u\omega}$	$-28.6\,kg/m$
Z_{uu}	$-6.15\,kg/(m.rad)$
$Z_{\omega\|\omega\|}$	$-131\,kg/m$
$Z_{q\|q\|}$	$-0.632\,kg.m/rad^2$
$M_{\dot{q}}$	$-4.88\,kg.m^2/rad$
$M_{\dot{\omega}}$	$-1.93\,kg.m$
M_{uq}	$-2\,kg.m/rad$
$M_{u\omega}$	$24kg$
M_{uu}	$-6.15\,kg/rad$
$M_{\omega\|\omega\|}$	$-3.18\,kg$
$M_{q\|q\|}$	$-188\,kg.\ m^2/rad^2$

Source: Authors.

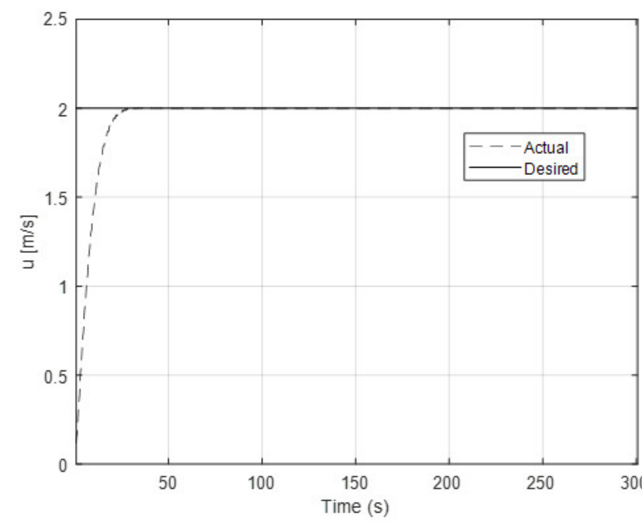
The simulations are performed using MATLAB software.

First, results for an ideal model (no perturbations or uncertainties) are given.

Figure 2 shows the first result, the transition velocity u, and how it reaches the reference value in a short time.

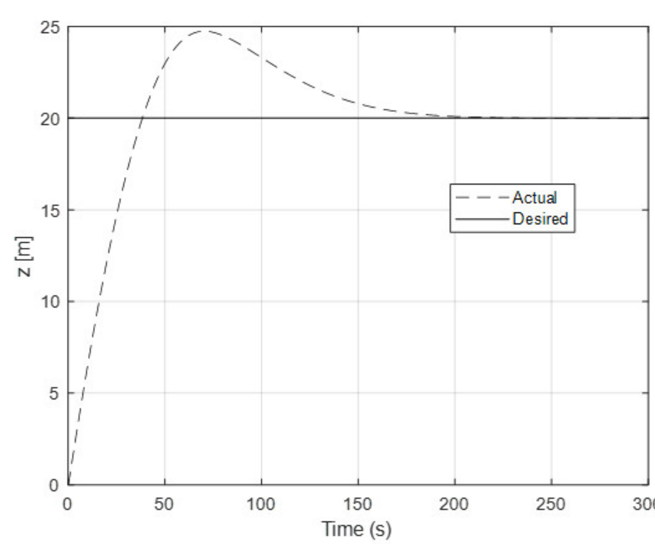
Figure 3 shows the vehicle's depth response, reaching the reference level with very little overshoot.

Figure 2: The actual and desired forward velocity response.



Source: Authors.

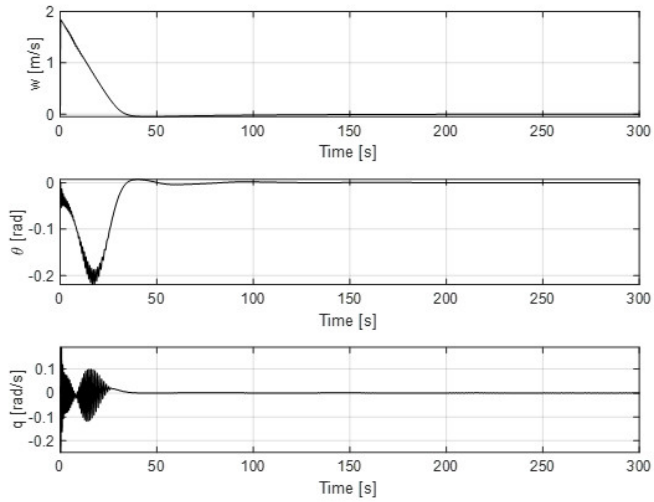
Figure 3: The actual and desired depth response .



Source: Authors.

Figure 4 shows the behavior of the remaining state variables, where it can be seen that these variables reach a steady state and there is no instability.

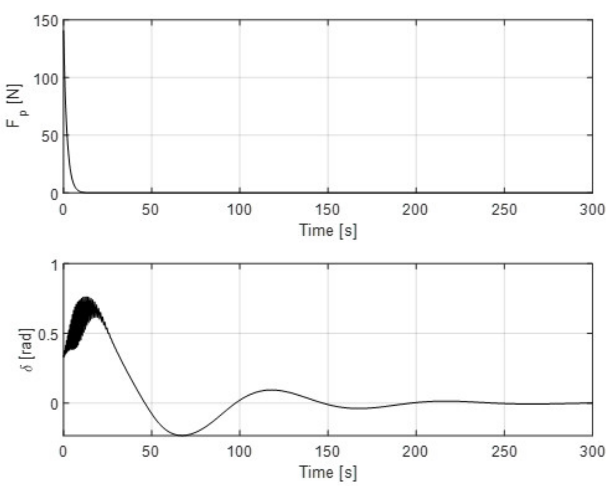
Figure 4: States$w$, θ and q responses.



Source: Authors.

Figure 5 shows the response of the obtained control laws applied to the system.

Figure 5: Control laws responses.

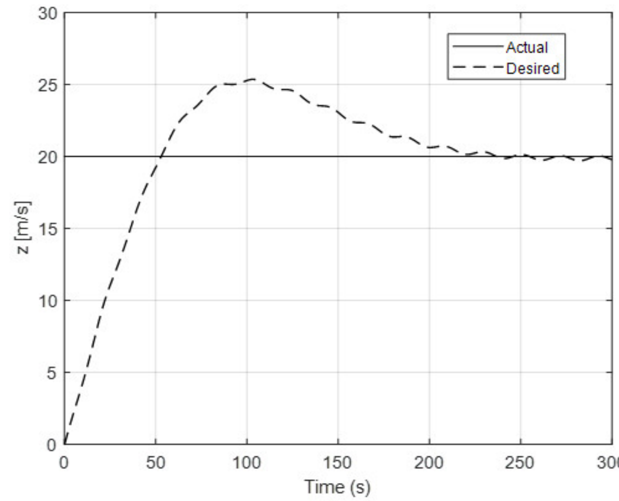


Source: Authors.

In order to evaluate the robustness and performance of the designed control scheme, disturbances are set to $Z_H = 0.3sin(t)$ and $M_P= 0.2cos(1.5t)$. Moreover, the system parameters are changed, a 20% variations from the nominal values are supposed.

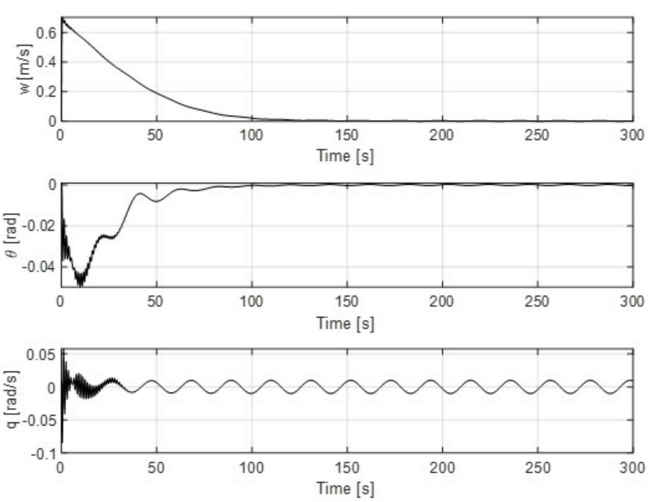
Figure 6 illustrates how the z-depth reaches the reference with satisfactory performance despite the perturbations and uncertainties in the parameters. Again, the responses of the remaining state variables are shown in Figure 7, demonstrating that the proposed control scheme is resilient to disturbances and limited uncertainties. The responses obtained from the applied control laws are shown in Figure 8, where it can be clearly observed that their magnitudes remain consistent with those obtained in the absence of disturbances.

Figure 6: The actual and desired depth response in presence of disturbances and parameters uncertainties.



Source: Authors.

Figure 7: States w, θ and q responses in presence of disturbances and parameters uncertainties.



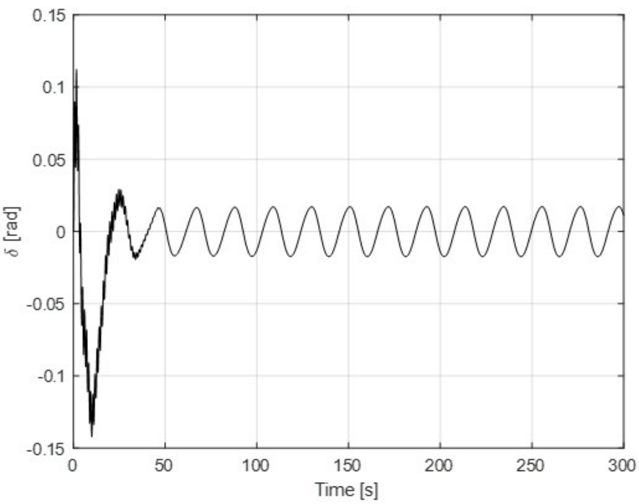
Source: Authors.

Conclusions.

In this paper, a novel terminal sliding mode control-based funnel control scheme using nonlinear observer for trajectory tracking of the underwater vehicle system is proposed.

This controller aims to make the AUV move up and down along a planned path. Additionally, the vehicles forward speed is also controlled. The proposed depth control can be used to reject disturbance because the behavior of an AUV is affected by unknown disturbance forces and moments. According to the simulation outcomes, the suggested control methods successfully tackle the problem of AUV trajectory tracking during

Figure 8: Control law responses in presence of disturbances and parameters uncertainties.



Source: Authors.

depth motion. Additionally, some simulation studies are shown to prove that these control schemes can handle disturbances and uncertainties within their limits.

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