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Depth Terminal Sliding Mode Technique Based-Funnel Control for Autonomous Underwater Vehicles

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ABSTRACT

This article proposes the design of a new funnel-based sliding mode controller for Autonomous Underwater Vehicle (AUV) depth control. The work also proposes a trajectory-tracking control scheme. The scheme is developed using the sliding mode control method and is robust to limited disturbances. Additionally, funnel variables are used to keep the tracking error within the funnel boundaries. Finally, the suggested control strategy has been validated by computer simulations. These simulation studies demonstrate the effectiveness of the proposed control strategy.

1. Introduction.

Nowadays, controlling Remotely Operated Underwater Vehicles (ROVs) and Autonomous Underwater Vehicles (AUVs), which are grouped according to their degree of energy and decision autonomy [1], is an important subject for in-depth study, given the vehicles' wide use in subsea missions such as deep-sea inspections, ocean mapping and several military applications, including detection, localisation and neutralisation.

Controlling AUVs is crucial to improving their efficiency and safety. However, designing their motion control algorithm is difficult because of their complex dynamics, which include nonlinear AUV dynamics, unmodelled dynamic effects, system uncertainties, and environmental disturbances.

Furthermore, standard AUV prototypes can't operate efficiently when the number of control inputs is smaller than the vehicle's degrees of freedom, indicating that ordinary linear controllers are insufficient for solving these problems.

In [2], the authors proposed a model reference adaptive control approach using hyperstability theory to control the depth of the REMUS autonomous underwater vehicle. Following this, the controller was improved using an artificial neural network. According to the results, the control was performed efficiently against parameter uncertainties and disturbances. Authors in [3] implemented an Artificial Bee Colony (ABC) algorithm to enhance the performance of a PID controller for depth regulation of an autonomous underwater vehicle (AUV). This approach improves stability and accuracy, even in difficult marine conditions. Based on the results, the ABC-based controller outperforms traditional methods and provides fast, precise depth tracking. In the literature [4], an innovative PID controller is employed to regulate the AUV's vertical-plane depth control and horizontal - plane motion-tracking control. The results demonstrate that the fuzzy self-adapting PID controller is considerably more resilient than the dual PID controller. It also shows that the controller can improve system performance compared to a traditionally tuned PID controller. However, PID cannot provide accurate control in the event of sea current disruptions. In [5], the authors presented the "Ocean Strider", a novel hybrid underwater vehicle. It is managed using two distinct techniques: a joystick for manual control and an au-

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tonomous control system that combines vision and ultrasonic sensors. The results indicate that the mechatronic design and control approaches are successful, demonstrating the practicality of the underwater hybrid vehicle for a wide range of maritime applications.

Additionally, the use of fast image sensors could be improved to enhance the vehicle's ability to identify objects. The study in [6] suggests using Physics - Informed Neural Networks (PINN) to optimise model predictive control (MPC) for an autonomous underwater vehicle, improving accuracy and efficiency in complex maritime conditions. The adaptive nonsingular integral terminal sliding mode fault-tolerant control for autonomous underwater vehicles is presented in [7], ensuring robustness and resilience in the face of problems. To track the trajectory of autonomous underwater vehicles (AUVs) in the presence of disturbances, researchers in [8] present a PID controller with a fixed-time sliding mode. By enhancing tracking performance, this technique ensures fast convergence and greater resistance to external perturbations.

In [9], a second-order sliding-mode trajectory-tracking controller is considered for surface vehicles. It consists of an equivalent controller and a switching controller that reduces parameter uncertainties and mitigates external perturbations. The closed-loop system is exponentially stable in the presence of uncertain hydrodynamics and unknown disturbances. Nonlinear Lyapunov-based techniques are implemented in [10-11] to create trajectory-tracking controllers for underactuated vehicles. In [10], a cascade approach is used. The derived control law is of simple structure and ensures global exponential stability of the tracking error dynamics. A second advantage of the cascaded structure is that it splits the problem of stabilising the nonlinear tracking error dynamics into the problem of stabilising two linear systems. This observation makes gain tuning simpler and much more amenable to finite optimal control for finding suitable gains. In [11], the authors explain the behaviour of the ship model, which resembles but cannot be immediately brought into the class of chained systems due to its drift vector field. They then use a coordinate transformation to recast the model in a triangular-like form before applying a backstepping approach with an added integrator to create a tracking control law. This control law is intended to control the ship's position and heading angle while maintaining exponential stability on the reference trajectory. Experimental results reveal that the ship can exponentially converge to a neighbourhood of the reference path despite difficulties.

The Lyapunov direct method employed in [12] is used to address the trajectory-tracking control problem for underactuated vehicles. Using the Lyapunov direct method, the work proposes two control strategies to guarantee global tracking under specific persistent excitation conditions. The cascading structure inherent in ship dynamics is at the origin of these strategies. The study introduces two control algorithms that employ Lyapunov's direct approach to ensure global tracking under persistent excitation conditions. These techniques exploit the cascading nature of ship dynamics, offering simple, systematic methods. They enable underactuated ships to precisely follow their specified trajectories, even in the presence of environmental disturbances.

The study in [13] presents an adaptive trajectory-tracking control approach for underactuated AUVs, utilising neural networks. It addresses issues related to unknown dynamics and asymmetric actuator saturation, ensuring precise tracking despite these uncertainties.

A control strategy for autonomous underwater vehicles that combines a Finite - Time Extended State Observer (FTE - SO) with Non - singular Fast Terminal Sliding Mode Control (NFT-SMC) is presented in [14]. The extended state observer estimates unknown dynamics and disturbances, while the fast terminal sliding mode ensures finite-time convergence, making it a powerful solution for control challenges in complex marine environments.

In this paper, the researchers introduce a falsification method that uses real-time input-output data for adaptive control. The approach aims to optimise the estimation of unknown parameters in the AUV. These estimates are then integrated into the controller to achieve accurate trajectory tracking. A comparative analysis demonstrates the method's efficiency [15]. This literature [16] examines modelling techniques and control strategies for unmanned underwater vehicles (UUVs). The study uses neural networks to estimate the system's complex dynamics and applies adaptive control to maintain precise positioning. The approach aims to improve trajectory performance and accuracy in dynamic marine environments.

In [17], a fuzzy dynamic surface control approach was developed to address the 3D trajectory-tracking problem for an underactuated AUV, accounting for uncertainties integrated into the model and time-varying disturbances. A predictive control method based on a variable fuzzy predictor was proposed to address dynamic trajectory-tracking supervision of an AUV in a three-dimensional underwater environment [18]. The authors in [19] propose a novel funnel-based sliding mode controller for depth regulation of Autonomous Underwater Vehicles in the vertical plane. The approach integrates a variable funnel control mechanism with sliding mode control to enhance tracking accuracy and transient performance.

In addition to transient responses, the steady - state error can also be investigated using Lyapunov analysis across all approaches. Lately, a novel algorithm has been considered to enhance and shape the transient response of 'classes' of nonlinear systems. This algorithm is designed to create a suitable funnel [20]. The concept ensures that a system's output follows a determined trajectory with specific performance criteria during the transient phase.

It employs control techniques to manage response time, overshoot, and settling times.

In a funnel-based control technique, the tracking error stays around the designed funnel. Therefore, the transient response characteristics can be tuned by selecting the funnel shape. Furthermore, funnel-based control is compatible with several well-known controllers, particularly the PI controller and sliding mode controller [21-22]. While numerous studies have been performed with funnel-based control, robust funnel-based control applied in autonomous underwater vehicle systems has not been studied. This paper discusses this issue. The major contribution of this work is the proposal to combine funnel-based control and sliding mode control for the AUV's trajectory-tracking problem in the vertical plane. New funnel variables are proposed and incorporated into the control architecture to keep the tracking error within a given boundary.

In addition, the control design of the vehicle's forward velocity is also taken into account. The Lyapunov stability hypothesis is used to analyse the stability of the designed controller. According to the simulation results, the proposed controller outperforms the traditional sliding mode controller in the presence of environmental disturbances and uncertainties.

The study is set up as described below. The second section provides a brief description of the AUV dynamic model and highlights the problem. In Section 3, a funnel control law for AUVs based on terminal sliding mode manifolds is derived. The given simulation results in Section 4 validate the proposed control mechanism. Finally, Section 5 presents an overview of this work's conclusion.

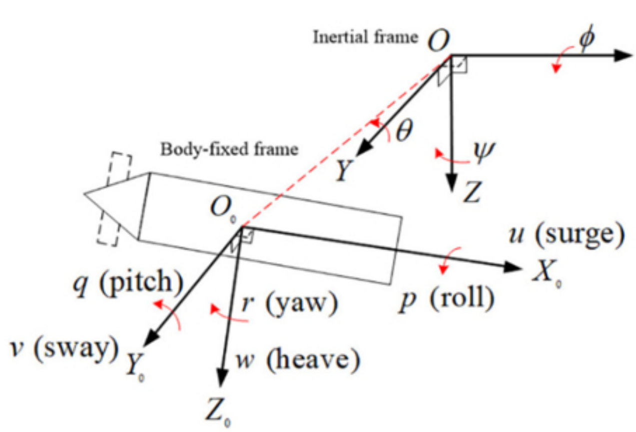
2. Mathematical Modelling of AUV.

The mathematical model of the underwater autonomous robot involves two (02) parts:

- Kinematic model, which describes the motion without taking into account the forces and torques that act on it; it only emphasises the geometrical aspects of motion (position, velocity and accelerations.
- A dynamic model considers the forces and moments to present the motion.

Underwater vehicle equations of motion are expressed with respect to an Earth-fixed frame and a body-fixed frame:

Figure 1: Time-varying hedge ratio estimated with VECH GARCH.



Source: Authors.

The Body-fixed position as measured in the Earth-fixed is:

$$\boldsymbol{\eta} = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}, \qquad \eta_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \qquad \eta_2 = \begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix} \tag{1}$$
$$\boldsymbol{\nu} = \dot{\boldsymbol{\eta}}$$

2.1. Kinematic Model.

Marine vehicles' kinematics are usually described in 6 degrees of freedom, including translation and rotation components, which are conveniently defined as: surge, sway, and heave for translation, describing the position of the robot, and pitch, yaw, and roll for rotation, describing the direction.

The relation linking the vehicle-fixed linear and angular velocity and the time derivative of the earth-fixed vehicle position coordinates is:

$$\dot{\eta} = J_c(\eta_2)\, v \tag{2}$$

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \begin{bmatrix} J_{c1}(\eta_2) & 0_{3*3} \\ 0_{3*3} & J_{c2}(\eta_2) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{3}$$

with

$$J_{c1}(\eta_2) = \begin{bmatrix} c\theta c\Psi & s\theta s\phi c\Psi - s\Psi c\phi & s\theta c\phi c\Psi + s\Psi s\phi \\ c\theta s\Psi & s\theta s\phi s\Psi + c\Psi c\phi & s\theta c\phi s\Psi - c\Psi s\phi \\ -s\theta & c\theta c\phi & c\theta c\phi \end{bmatrix} \tag{4}$$

$$J_{c2}(\eta_2) = \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\phi/c\theta \end{bmatrix} \tag{5}$$

where:

$J_{c1}(\eta_2)$: the transformation matrix of the linear velocity.

$J_{c2}(\eta_2)$: the transformation matrix of the angular velocity.

2.2. Dynamic Model.

The underwater vehicle is rigid, and its 6-DOF rigid-body dynamics obey either the Newton-Euler formulation (Newton's second law) or the Lagrangian Formulation (momentum and energy conservation) [2].

$$\begin{cases} m * \gamma = \sum F \\ I * \ddot{\alpha} = \sum M \end{cases} \tag{6}$$

Where:

F: the net external force applied to the vehicle.

m: the mass of the vehicle

γ : the acceleration of its mass centre with respect to an inertial frame,

M: the net external moment acting on the vehicle at the centre of mass,

I: the inertia tensor about the vehicle's mass centre.

$\ddot{\alpha}$: is the vehicle's angular acceleration.

The nonlinear vehicle equation of motion is written as:

$$\begin{cases} \dot{\eta} = J_c(\eta_2)\, v \\ m\dot{v} + C(v)\, v + D(v)\, v + g(\eta) = \Gamma \end{cases} \tag{7}$$

We focus our attention on the dive plane with the horizontal plane-control surfaces at zero to simplify the control law design approach. In the body-fixed coordinate frame, the heave and pitch equations of motion for the vehicle are as follows:

$$\begin{aligned} m(\dot{u}+qw) &= X_{qq}q^2 + X_{\dot{u}}\dot{u} + X_{wq}wq + X_{q\delta}uq\delta \\ &+X_{ww}w^2 + X_{w\delta}uw\delta + X_{\delta\delta}u^2\delta^2 \\ &-(W-B)\sin\theta + F_p \end{aligned} \tag{8}$$

$$\begin{aligned} m(\dot{w}-uq) &= z_{\dot{q}}\dot{q} + z_{\dot{w}}\dot{w} + z_{uq}uq + z_{uw}uw \\ &+z_{uu}u^2\delta + (W-B)\cos\theta + z_{w|w|}w|w| \\ &+z_{q|q|}q|q| + z_H \end{aligned} \tag{9}$$

$$\begin{aligned} I_{yy}\dot{q} &= M_{\dot{q}}\dot{q} + M_{\dot{w}}\dot{w} + M_{uq}uq + M_{uw}uw \\ &+M_{uu}u^2\delta - (z_G W - z_B B)\sin\theta - (x_G W - x_B B)\cos\theta \\ &+M_{w|w|}w|w| - M_{q|q|}q|q| + M_p \end{aligned} \tag{10}$$

$$\dot{\theta} = q \tag{11}$$

$$\dot{z} = -usin\theta + wcos\theta \tag{12}$$

Where u is the vehicle's forward velocity, ω is the heave velocity, θ is the pitch angle, q is the pitch angle velocity, z is the vehicle's depth, δ is the control fin angle, FP is the propulsion force which controls the forward velocity, m is the mass of the vehicle, I_{yy} is the moment of inertia of the vehicle about the pitch axis, W denotes the vehicle's weight and B is the vehicle buoyancy. $Z_{\dot{q}}$, Z_{uq}, $Z_{\dot{\omega}}$, $M_{\dot{q}}$, etc., are the hydrodynamics parameters. Finally, M_P and Z_H represent the cross-flow drag terms and are considered as disturbances.

Establishing the state vector

$$x = [x_1,\ x_2,\ x_3,\ x_4,\ x_5]^T \ = [u,\ \omega,\ q,\ \theta,\ z]^T,$$

The control vector $v = [F_P,\quad \delta]^T$ and the inertial matrix.

$$\mathbf{M} = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 & 0 & 0 \\ 0 & m - Z_{\dot{w}} & -Z_{\dot{q}} & 0 & 0 \\ 0 & -M_{\dot{w}} & I_{yy} - M_{\dot{q}} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

The vehicle dynamics are expressed in state space forms (1)

$$M\dot{x} = f_x(x,t) + g_x(x,v) \tag{14}$$

Or

$$\dot{x} = f(x,t) + g(x,v) \tag{15}$$

With outputs:

$$y_1 = u \tag{16}$$

$$y_2 = z \tag{17}$$

$$f(x,t) = M^{-1}f_x(x,t)$$

And

$$g(x,v) = M^{-1}g_x(x,v) \tag{18}$$

3. Control of the AUV.

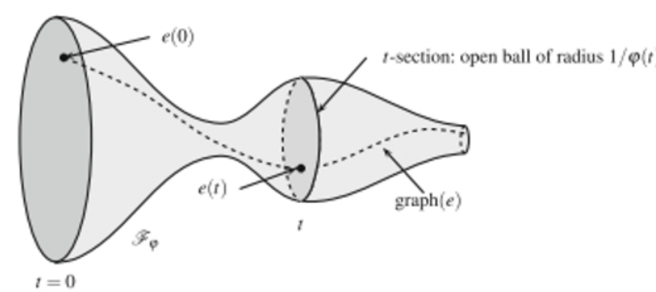
A combination of sliding mode control and funnel control is used to control the AUV.

3.1. Funnel Control Approach.

Funnel control is a control approach employed in many applications. This approach aims to ensure time-varying reference signal tracking with defined transient accuracy, meaning that the tracking error remains within certain limitations. The advantages of this approach are simplicity, robustness, and error limitation [21].

To facilitate this process, achieving the desired transient behaviour and precision requires an adaptive gain in the error feedback [24].

Figure 2: Performance funnel [23].



Source: Authors.

$$u(t) = -k(t)e(t) \tag{19}$$

With

$$e(t) = y(t) - y_{ref}(t)$$

k(t) is determined adaptively using a predetermined error bound that changes over time.

$$k(t) = \frac{\varphi(t)}{1-\varphi(t)|e(t)|} = \frac{1}{\Psi(t)-|e(t)|} \tag{20}$$

Where $\psi(t) = \frac{1}{\phi(t)}$ is the boundaries of the funnel, φ(t) is a following class's function:

$$\Phi := \left\{\varphi(t) \in W^{1,\infty}(\mathbb{R}_+,\mathbb{R}_+) \,\middle|\, \forall\, t > 0,\ \varphi(t) > 0\right\}$$

And

$$\liminf_{\tau\to\infty} \varphi(\tau) > 0, \quad \forall\tau > 0 : \left\{\varphi^{-1}(\cdot) \text{ is globally Lipschitz}\right\}.$$

Where $w^{(1,\infty)}$ (R_+,R+) represents a class of functions with bounded derivatives. We consider F_φ that describes the funnel's performances:

$$F_\varphi := \{(t,e) \in \mathbb{R}_{\geq 0} \times \mathbb{R} \mid |e(t)| < \Psi(t)\} \tag{21}$$

The system's high-gain characteristic ensures that the error magnitude decreases.

3.1.1. Velocity Control.

Consider the output variable $x_1 = y_1 = u$ and the constant reference u_d. Because x_1 has relative degree one, a sliding manifold is designed as:

$$\sigma = x_1 - u_d \tag{22}$$

With time derivative:

$$\dot{\sigma} = f_u(x, t, \delta) + g_u F_p \tag{23}$$

The sliding mode manifold S0 is designed as follows:

$$S_0 = \frac{\sigma(t)}{F_\varphi(t) - |\sigma(t)|} \tag{24}$$

With:

$$\sigma = x_1 - u_d \tag{25}$$

$$F_\varphi(t) = \gamma_0 e^{-a_0 t} + \gamma_\infty \tag{26}$$

When:

$$\gamma_0 \geq \gamma_\infty > 0, \qquad a_0 > 0, \qquad \gamma_\infty = \lim_{t\to+\infty} F_\varphi(t)$$

The derivative of (24) can be calculated as:

$$\dot{S_0} = F_\varphi\, \Phi_F \dot{\gamma} - \dot{F_\varphi} \Phi_F\, \gamma \tag{27}$$

$$\Phi_F = \frac{1}{(F_\varphi(t) - |\sigma(t)|)^2}$$

$$\dot{S_0} = F_\varphi\, \Phi_F \left[f_u(x, t, \delta) + g_u F_p\right] - \dot{F_\varphi} \Phi_F\, \gamma \tag{28}$$

To make S_0 converge to zero within a finite time the non-singular terminal sliding mode manifold is developed as follows:

$$\beta \left|\dot{S_0}\right|^{q/p} sign(S_0) + S_0 = 0 \tag{29}$$

Where $\beta > 0$; p and q are positive add integers with $p < q$. Substituting (2) into (4), the controller is designed as:

$$F_p = \frac{1}{g_u}\left[\frac{\dot{F_\varphi}}{F_\varphi}\gamma - \frac{1}{F_\varphi \Phi_F}\frac{1}{\beta}|S_0|^{q/p}\ \mathrm{sign}(S_0) - \dot{u}_d - f_u(x, t, \delta)\right] \tag{30}$$

Theorem 1: Consider the sub-system (15-16) with the control law (30), then the tracking error σ is bounded.

Proof: Allow the subsequent Lyapunov function:

$$V_1 = \frac{1}{2} {S_0}^2 \tag{31}$$

Differentiating (31), we have:

$$\dot{V_1} = S_0 \dot{S_0} \tag{32}$$

$$\dot{V_1} = S_0\left[F_\varphi\, \Phi_F \left[f_u(x, t, \delta) + g_u F_p\right] - \dot{F_\varphi} \Phi_F\, \gamma \right] \tag{33}$$

Substituting (5) into (8), we obtain:

$$\dot{V_1} \leq -\frac{1}{\beta}|S_0|^{(p+q)/q} \leq 0 \tag{34}$$

Inequality (34) implies that S_0 is bounded then the subsystem's stability (15-16) with the control law (30) has been proved.

We require the following lemma in order to enhance the finite time convergence.

Lemma 1 assumes that there exists a continuous positive definite function V(t) satisfying the following inequality:

$$\dot{V}(t) + nV^\gamma(t) \leq 0, \forall\, t > t_0 \tag{35}$$

$$V(t) \equiv 0, \forall\, t \geq t_s$$

Where t_s is given by:

$$t_s \leq t_0 + \frac{V^{1-\gamma}(t_0)}{n(1-\gamma)} \tag{36}$$

We have:

$$\dot{V_1} \leq -\frac{1}{\beta}|S_0|^{(p+q)/q} \tag{37}$$

$$\dot{V_1} \leq -\frac{1}{\beta}(2)^{(p+q)/(2q)} \left(\frac{1}{2}{S_0}^2\right)^{(p+q)/(2q)} \tag{38}$$

$$\dot{V_1} \leq -\frac{1}{\beta}(2)^{(p+q)/(2q)}\, {V_1}^{(p+q)/(2q)} \tag{39}$$

$$\dot{V_1} \leq -k_1\, {V_1}^{k_2} \tag{40}$$

Where:

$$k_1 = \frac{1}{\beta} 2^{(p+q)/(2q)}; \tag{41}$$

$$k_2 = (p+q)/(2q) \tag{42}$$

From these, we can obtain:

$$\dot{V_1} + k_1\, {V_1}^{k_2} \leq 0 \tag{43}$$

Lemma (1) allows to determine the terminal sliding manifold's ability to converge to the equilibrium point within a finite time t_1 given by: $t_1 = \frac{V^{1-K_2(t_0)}}{k_1(1-K_2)}$

3.1.2. Depth Control.

The objective is to obtain a constant reference zd for the variable. Taking into consideration the second output variable $\overline{x}_2 = y_2 = z$, and introducing a new variable, $\overline{x}_3 = \dot{\overline{x}}_2$, for its time derivative, the depth subsystem is:

$$\dot{\overline{x}}_2 = -usin\theta + wcos\theta = \overline{x_3} \tag{44}$$

$$\dot{\overline{x}}_3 = -uqcos\theta - wqsin\theta + cos\theta(f_w(x, t) - g_w(x)\delta \tag{45}$$

Defining the new variables $\overline{x}_4 = q - 10w$ and $\overline{x}_5 = \theta$, the complete diffeomorphism results in

$$\overline{x} = \Phi(x) = \begin{bmatrix} u \\ z \\ -usin\theta + wcos\theta \\ q - 10w \\ \theta \end{bmatrix} \tag{46}$$

Where the form of the inverse transformation is

$$x = \Phi^{-1}(\overline{x}) = \begin{bmatrix} \overline{x}_1 \\ \frac{\overline{x}_3+\overline{x}_1 sin\overline{x}_5}{cos\overline{x}_5} \\ \overline{x}_4 + 10\left(\frac{\overline{x}_3+\overline{x}_1 sin\overline{x}_5}{cos\overline{x}_5}\right) \\ \overline{x}_5 \\ \overline{x}_2 \end{bmatrix} \quad (47)$$

The whole subsystem for depth control is:

$$\dot{\overline{x}}_2 = \overline{x}_3 \quad (48)$$

$$\dot{\overline{x}}_3 = \overline{f}_3(\overline{x}, t) - \overline{g}_3(\overline{x})\delta \quad (49)$$

$$\dot{\overline{x}}_4 = \overline{f}_4(\overline{x}_1, \overline{x}_4, \overline{x}_5, t) + \overline{g}_4(\overline{x})\overline{x}_3 \quad (50)$$

$$\dot{\overline{x}}_5 = \overline{f}_5(\overline{x}_1, \overline{x}_4, \overline{x}_5) + \overline{g}_5(\overline{x})\overline{x}_3 \quad (51)$$

Where:

$$\overline{f}_3(x, t) = -uqcos\theta - \omega q sin\theta + cos\theta\, f_\omega(x, t)$$

And

$$\overline{g}_3(x) = cos\theta\, g_\omega(x)$$

The second sliding manifold S_2 is formulated as:

$$s_2 = \dot{\sigma}_2 + \alpha_2\sigma_2 \quad (52)$$

$$(\alpha_2 > 0)$$

Where:

$$\sigma_2 = \frac{e_z}{F_{\phi_z} - |e_z|} \quad (53)$$

By deriving it in terms of time:

$$\dot{\sigma}_2 = F_{\varphi_2}\varphi_{F_2}\dot{e}_z - \dot{F}_{\varphi_2}\varphi_{F_2}e_z \quad (54)$$

With:

$$\varphi_{F_2} = \frac{1}{\left(F_{\varphi_2} - |e_z|\right)^2} \quad (55)$$

Deriving σ_2 for a second time:

$$\ddot{\sigma}_2 = F_{\varphi_2}\varphi_{F_2}\ddot{e}_z + \Gamma(x, e, t) \quad (56)$$

Where:

$$\Gamma(x, e_z, t) = F_{\varphi_2}\dot{\varphi}_{F_2}\dot{e}_z + \dot{F}_{\varphi_2}\varphi_{F_2}\dot{e}_z - \dot{F}_{\varphi_2}\dot{\varphi}_{F_2}e_z - \dot{F}_{\varphi_2}\varphi_{F_2}\dot{e}_z - \ddot{F}_{\varphi_2}\varphi_{F_2}e_z \quad (57)$$

The derivative of s_2 is:

$$\dot{s}_2 = \ddot{\sigma}_2 + \alpha_2\dot{\sigma}_2 \quad (58)$$

$$\dot{s}_2 = F_{\varphi_2}\varphi_{F_2}\left(f_3(\overline{x}, t) - g_3(\overline{x})\delta - \ddot{z}^d\right) + \Gamma(x, e_z, t) + \alpha_2\dot{\sigma}_2 \quad (59)$$

$$\dot{s}_2 = F_{\varphi_2}\varphi_{F_2}\left(\Sigma(\overline{x}, e_z, t) - g_3(\overline{x})\delta - \ddot{z}^d\right) + \alpha_2\dot{\sigma}_2 \quad (60)$$

The nonlinear function $\Sigma(\overline{x}, t)$ is:

$$\Sigma(\overline{x}, e_z, t) = f_3(\overline{x}, t) + \frac{\Gamma(\overline{x}, e_z, t)}{F_{\varphi_2}\varphi_{F_2}} \quad (61)$$

To ensure that s_2 converge to zero in a finite time, the sliding mode collector is established as follows:

$$s_2 + \beta_2|\dot{s}_2|^{q_2/p_2} sign(s_2) = 0 \quad (62)$$

$$(\beta_2, q_2, p_2 0 \text{ and } p < q)$$

The control law is designed as:

$$\delta = -\frac{1}{g_3(\overline{x})}\left[-\ddot{z}_d + \sum(\overline{x}, e_z, t) + \frac{1}{F_\varphi\varphi_{F_2}}\left(\alpha_2\dot{\sigma}_2 + \frac{1}{\beta_2}|S_2|^{q_2/p_2}\,\text{sign}(S_2)\right)\right] \quad (63)$$

Theorem 2:

For the subsystem expressed by (48-51) and using the control in (63), the vehicle depth will converge to their reference in finite time.

Proof:

By choosing the Lyapunov function as follows:

$$V_2 = \frac{1}{2}{s_2}^2 \quad (64)$$

By deriving it in terms of time:

$$\dot{V}_2 = s_2\dot{s}_2 \quad (65)$$

By replacing $\dot{s}_2$:

$$\dot{V}_2 = S_2\left[F_{\varphi_2}\varphi_{F_2}\left(\sum(\overline{x}, e_z, t) - g_3(\overline{x})\delta - \ddot{z}_d\right) + \alpha_2\dot{\sigma}_2\right] \quad (66)$$

Substituting (63) into (66) yield:

$$\dot{V}_2 \le -\frac{1}{\beta_2}|S_2|^{(p_2+q_2)/q_2} \le 0 \quad (67)$$

$$\dot{V}_2 \le -\frac{1}{\beta_2}2^{(p_2+q_2)/(2q_2)}{V_2}^{(p_2+q_2)/(2q_2)} \quad (68)$$

$$\dot{V}_2 \le -K_3{V_2}^{K_4} \quad (69)$$

Where:

$$K_3 = \frac{1}{\beta_2}2^{(p_2+q_2)/(2q_2)} \quad (70)$$

And:

$$K_4 = (p_2 + q_2)/(2q_2) \quad (71)$$

then, we obtain:

$$\dot{V}_2 + K_3{V_2}^{K_4} \le 0 \quad (72)$$

hence, it possible to conclude that the sliding manifold S_2 is able to converge to the equilibrium point within a finite time t_2 given by

$$t_2 = \frac{{V_2}^{(1-K_4)}(t_0)}{K_3(1 - K_4)} \quad (73)$$

4. Simulation Results.

To validate and enhance the performance of the suggested control scheme, numerical simulations were performed using an AUV model [11]. The vehicle model parameters are provided in Table 1. The forward speed u reference is set to 3 m/s and the depth z reference is set to 25 m.

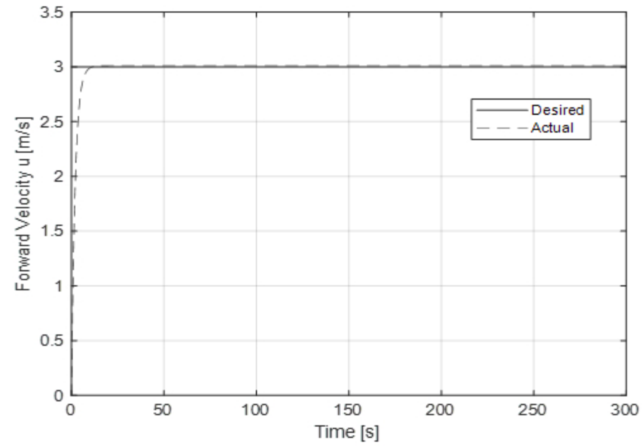
Table 1: AUV model parameters and controller parameters design.

Parameter	value
W	$299\ N$
m	$30.48\ kg$
I_{yy}	$3.45\ kg.m^2$
z_G	0
z_B	$-0.0196\ m$
x_G	0
y_G	0
x_B	0
y_B	0
B	$306\ N$
X_{qq}	-1.5
$X_{\dot{u}}$	-16
$X_{\omega q}$	-20
$X_{q\delta}$	2.5
$X_{\omega\omega}$	17
$X_{\omega\delta}$	4.6
$X_{\delta\delta}$	1
$Z_{\dot{q}}$	$-1.93\ kg.m/rad$
$Z_{\dot{\omega}}$	$-35.5\ kg$
Z_{uq}	$-5.22\ kg/rad$
$Z_{u\omega}$	$-28.6\ kg/m$
Z_{uu}	$-6.15\ kg/(m.rad)$
$Z_{\omega\|\omega\|}$	$-131\ kg/m$
$Z_{q\|q\|}$	$-0.632\ kg.m/rad^2$
$M_{\dot{q}}$	$-4.88\ kg.m^2/rad$
$M_{\dot{\omega}}$	$-1.93\ kg.m$
M_{uq}	$-2\ kg.m/rad$
$M_{u\omega}$	$24kg$
M_{uu}	$-6.15\ kg/rad$
$M_{\omega\|\omega\|}$	$-3.18\ kg$
$M_{q\|q\|}$	$-188\ kg.\ m^2/rad^2$

Source: Authors.

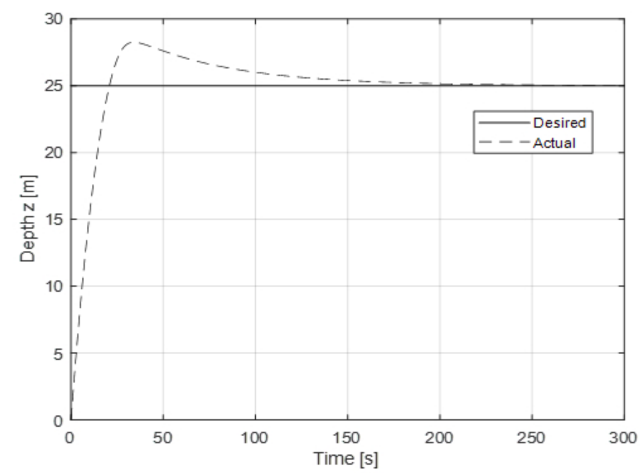
Simulations are performed using MATLAB Software. First, results for an ideal model (without disturbances or uncertainties) are given. Figure 3 illustrates the first output, the forward velocity u performance and how it approaches the reference value in a short amount of time. Figure 4 shows the vehicle's depth response, reaching the reference level with minimal overshoot.

Figure 3: The actual and desired forward velocity response.



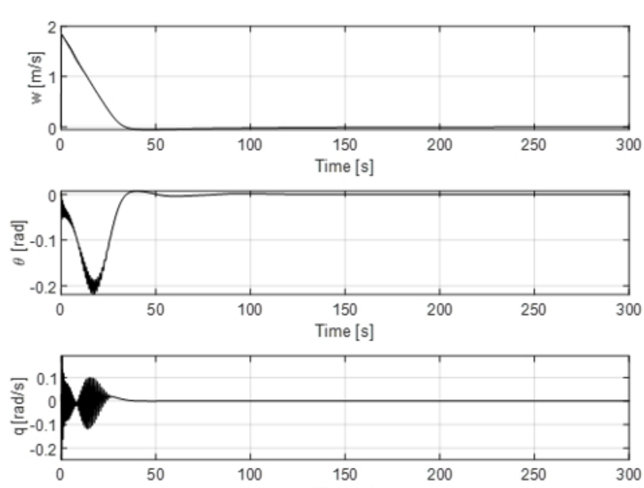
Source: Authors.

Figure 4: The actual and desired depth response.



Source: Authors.

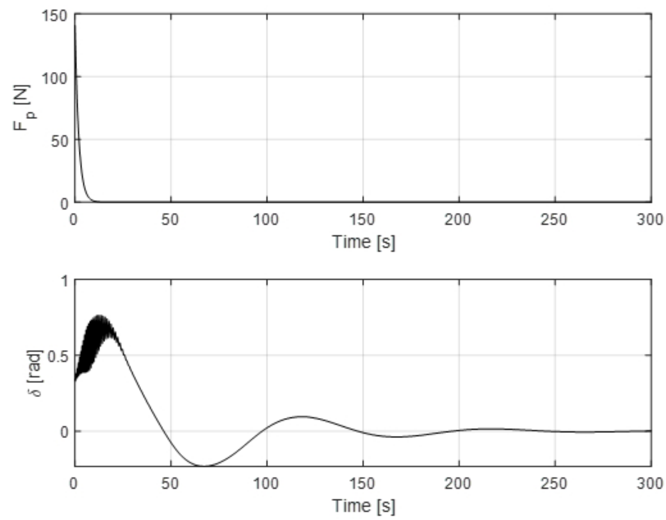
Figure 5: States$w$, θ and responses.



Source: Authors.

Figure 5 demonstrates how the remaining state variables behave, showing how these variables attain a steady state and there is no instability.

Figure 6: Control laws responses.

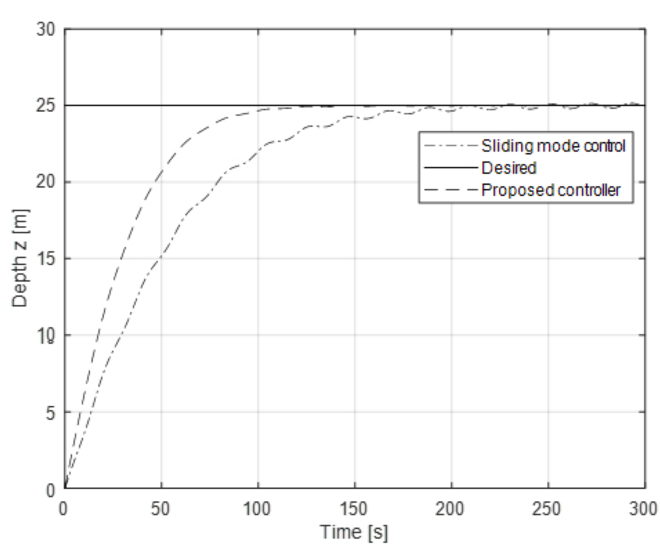


Source: Authors.

Figure 6 illustrates the response of the obtained control laws applied to the system.

To evaluate the performance and resilience of the proposed control scheme, disturbances are set to $Z_H = 0.3sin(t)$ and $M_P = 0.2cos(1.5t)$. Moreover, the system parameters are changed, 20% variations from the nominal values are supposed.

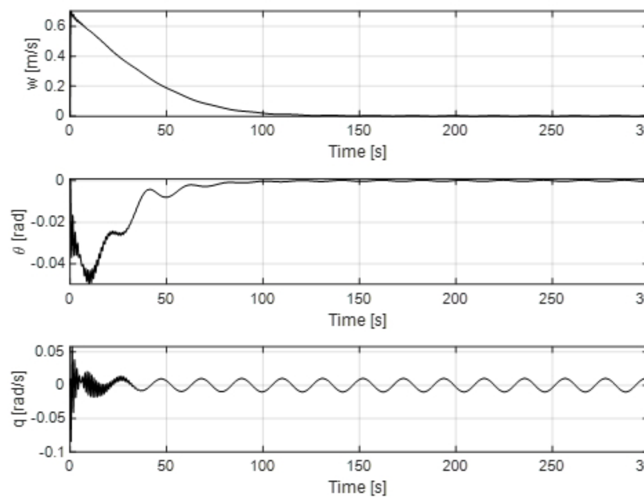
Figure 7: The actual and desired depth response in the presence of uncertainty in the parameters and disruptions.



Source: Authors.

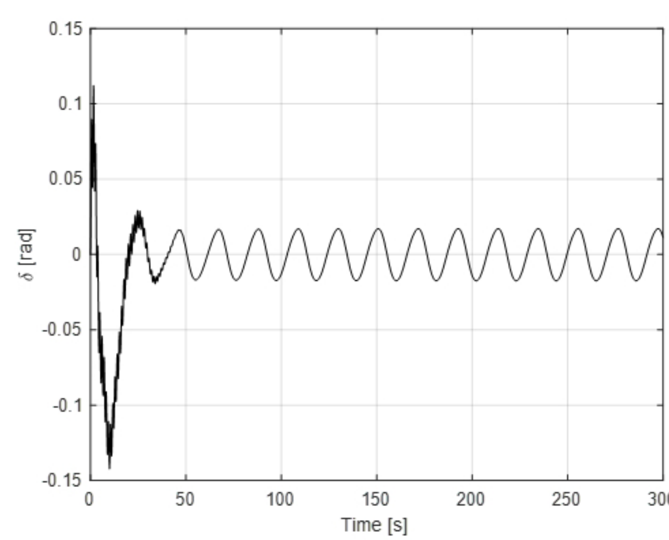
Figure 7 shows how the depth z, in spite of the disturbances and uncertainties in the parameters, achieves the reference with a suitable efficiency. Once again, the responses of the remaining state variables are shown in Figure 8, demonstrating that the proposed control scheme is resilient to disturbances and bounded uncertainties. The responses derived from applied control laws are depicted in Figure 9, wherein it is clear that their magnitude remains consistent with the results produced without any disruptions.

Figure 8: States w, θ and q responses in presence of disturbances and parameters uncertainties.



Source: Authors.

Figure 9: Control law responses in presence of disturbances and parameters uncertainties.



Source: Authors.

Conclusions.

Focusing on Autonomous Underwater Vehicle trajectory-tracking control. In this work, we discuss the control of an AUV system using a new terminal sliding mode control-based funnel control technique. The primary purpose of this controller is to ensure the AUV's vertical position follows a predetermined trajectory. Furthermore, a control design for the vehicle's forward velocity is considered. Since the behaviour of an AUV is influenced by unknown disturbance forces and moments, the proposed depth control can be used to reject disturbances. The simulation results show that the proposed control strategies successfully handle the trajectory-tracking issue in the depth motion of AUVs Furthermore, simulation studies demonstrate that these control methods are robust to bounded disturbances and parameter uncertainties.

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