



## The Role of Graph Theory in Maritime Education and Research

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### ABSTRACT

In this research article we focused on Graph Theory, a foundational branch of discrete mathematics, which offers a robust abstract framework for modeling complex systems through vertices and edges. Within maritime sector education and research, it enables the conceptual analysis of interconnected networks, including port connectivity, ship subsystems, shipping logistics, and global trade routes, without relying on empirical data. This paper examines the theoretical role of graph theory in maritime universities, with particular emphasis on institutions such as Gdynia Maritime University in Poland and Shanghai Maritime University in China. It also explores the pedagogical integration of graph theory into maritime curricula, highlighting its effectiveness in cultivating analytical and deductive reasoning skills essential for addressing maritime challenges. The study further investigates theoretical applications across maritime transportation networks, naval architecture, and shipping logistics, through key concepts such as connectivity, centrality, cycles, domination, weighted graphs, and minimum spanning trees. Recognizing the limitations of static graph models in representing the dynamic, uncertain, and multifaceted nature of maritime systems, this paper proposes advanced theoretical extensions – namely temporal graphs, fuzzy graphs, and hypergraphs – to enhance modeling capacity and conceptual flexibility. By prioritizing abstract reasoning over computational implementation, this work underscores the enduring significance of graph theory in preparing maritime students and researchers to confront contemporary challenges in global trade, sustainable ocean economies, and resilient maritime infrastructure.

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### 1. Introduction.

Maritime universities, including Gdynia Maritime University in Poland and Shanghai Maritime University in China, enhance education and research in naval architecture, maritime engineering, shipping, and ocean sciences. They address the complexities of global trade, where over 90% of goods are transported by sea. These disciplines deal with intricate networks, with ports linked by shipping routes, ships having interdependent subsystems, and logistics chains that span multiple transport modes.

To model these systems abstractly, maritime education and research need mathematical frameworks that represent structural relationships without empirical data. Graph theory serves as a fundamental tool for understanding pairwise relationships in complex maritime systems.

Graph theory began with the famous 1736 solution to the Seven Bridges of Königsberg problem. The Königsberg problem visualizes entities as vertices and relationships as edges. This visualization enables the theoretical study of network properties such as connectivity and efficiency within maritime contexts. Ports can be viewed as vertices and sea routes as edges, allowing exploration of network resilience or optimization. Directed graphs represent one-way shipping lanes affected by ocean currents, whereas weighted graphs illustrate abstract costs such as distance or environmental impact, supporting theoretical analyses of system behavior. This abstraction empowers maritime researchers and students to hypothesize about network structures, such as the robustness of trade routes, without relying on

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numerical data.

Graph theory connects mathematics with maritime disciplines, fostering strong analytical frameworks. It is integrated into discrete mathematics and operations research courses, equipping students to model systems like port networks in an abstract manner, thus improving their analytical skills. In research, it aids in understanding challenges such as network vulnerabilities by using properties like centrality and domination. This reflects the mission of maritime universities to prepare students and researchers for an industry dealing with globalization, environmental, and technological challenges. This paper examines the theoretical role of graph theory, focusing on its basic principles, pedagogical integration, and applications in transportation, naval architecture, and logistics, concluding with its theoretical challenges and extensions.

## 2. Fundamentals of Graph Theory in Maritime Contexts.

Graph theory lays the groundwork for modeling pairwise relationships in maritime systems through graphs. Formally, a graph  $G = (V, E)$  consists of a vertex set  $V$ , which denotes entities, and an edge set  $E$ , which signifies the connections between pairs of vertices. In maritime education and research, like those at Gdynia Maritime University and Shanghai Maritime University, graphs abstractly represent complex networks of ports, ships, and subsystems, allowing for theoretical exploration of structural properties without relying on empirical data. Vertices can denote ports, waypoints, or ship components; edges may represent sea routes, communication lines, or dependencies, promoting a conceptual analysis of maritime issues.

Various forms of graphs offer distinct theoretical insights for maritime applications. Undirected graphs illustrate symmetric relationships, such as bidirectional shipping routes between ports, where edges imply mutual access. Directed graphs, or digraphs, detail asymmetric relationships, like one-way shipping lanes influenced by ocean currents or regulations, which is critical for theorizing maritime navigation. Weighted graphs assign values to edges that represent theoretical metrics such as distance, time, or environmental cost, which helps researchers conceptualize efficiency in maritime networks. For example, a weighted edge between two ports could reflect navigational effort, enabling theoretical studies on route optimization. Multigraphs, which allow multiple edges between vertices, can model parallel connections, such as multiple shipping routes between ports, improving flexibility in theoretical representations.

Key graph properties are particularly significant for maritime systems. Connectivity guarantees a path between any pair of vertices, modeling seamless networks like global trade systems where every port is theoretically reachable. A disconnected graph highlights vulnerabilities, such as isolated ports, guiding discussions on network robustness. Cyclic graphs, or closed paths that return to the starting vertex, represent alternative routes and are essential for theorizing fault-tolerant systems. A cyclic graph in a port network ensures that alternate routes exist, enhancing resilience against disruptions. Centrality measures, including degree centrality (the number of edges connecting to a vertex) and betweenness centrality (vertices that

lie on many shortest paths), identify crucial nodes, such as major ports or key ship components. Domination, where a subset of vertices ensures every vertex is either in the set or adjacent to it, models minimal resource allocation, such as the placement of theoretical monitoring stations to cover a maritime network.

Additional graph-theoretic constructs broaden maritime applications. Planar graphs, which can be drawn without edge crossings, model layouts like port terminals or ship deck configurations, optimizing space conceptually. Bipartite graphs contain vertices divided into two sets to represent interactions, such as ship-port relationships, allowing for scheduling or resource allocation. Graph coloring assigns labels to vertices or edges to prevent conflicts, which models scheduling challenges such as non-overlapping port operations. These constructs offer a flexible toolkit for maritime scholars to model systems, enabling rigorous analysis in education and research. By focusing on structural properties, graph theory helps students and researchers conceptualize maritime networks, preparing them to tackle complex challenges through theoretical approaches.

## 3. Graph Theory in Maritime University Curriculum.

Graph theory is a fundamental part of the curricula at maritime universities such as Gdynia Maritime University and Shanghai Maritime University. It provides a theoretical base for modeling complex maritime systems, primarily focused on discrete mathematics and operations research courses. Graph theory gives students abstract tools to understand interconnected systems with vertices and edges, fostering analytical and deductive reasoning skills essential for maritime fields. By representing entities like ports, ships, or subsystems as graphs, students learn to transform real-world challenges into mathematical structures. This enables exploration of properties like connectivity and efficiency without empirical data reliance. This section discusses the educational role of graph theory, highlighting its integration into mathematics and interdisciplinary courses, its role in skill development, and its importance in preparing students to address maritime challenges conceptually.

In mathematics courses, graph theory is presented as a way to model pairwise relationships. Vertices signify entities (e.g., ports, ship components), while edges represent connections (e.g., sea routes, dependencies). Students learn to view port networks as graphs and examine properties like connectivity to theorize robust trade routes, where a connected graph ensures all ports are theoretically reachable. This abstraction promotes deductive reasoning, allowing students to think critically about network behavior and identify vulnerabilities in disconnected graphs without empirical limitations. Exercises may involve creating graphs to represent maritime networks, encouraging students to investigate structural properties like cycles, which symbolize redundancy critical for maintaining resilience, thus enhancing their ability to handle abstract maritime challenges.

The educational impact of graph theory extends to interdisciplinary courses in logistics, naval architecture, and transportation. In logistics courses, students use directed graphs to represent one-way shipping lanes affected by currents, theorizing efficient routing strategies. Cyclic paths represent al-

ternative routes, enabling students to explore fault-tolerant systems. In naval architecture, graphs depict subsystem dependencies, where vertices represent factors (e.g., electrical nodes) and edges represent connections (e.g., wiring). This allows students to theorize about system resilience through connectivity analysis. A connected graph shows that all parts are integrated, while a disconnected graph reveals potential failure points, helping students gain insights into robust design. These applications connect theoretical mathematics with maritime challenges, preparing students for complex systems.

Teaching graph traversal concepts such as paths and cycles is vital in maritime curricula, enabling students to develop efficient configurations. A path, defined as a series of edges linking vertices, may model flows like goods within a port network or signals in a ship's communication system. Cycles provide redundancy, essential for modeling resilient networks, like backup routes in logistics or alternative paths within ship subsystems. In operations research classes, students investigate concepts such as minimum spanning trees – acyclic subgraphs that connect all vertices with the least edge weight – allowing them to theorize efficient port connections while improving their analytical skills for maritime applications. Centrality measures may help students pinpoint critical nodes, like major ports or key components, and develop prioritization skills.

Integrating graph theory encourages interdisciplinary thinking, a key goal for maritime universities. Bipartite graphs show interactions between two sets of vertices to model ship-port relationships, aiding in theorizing scheduling or resource allocation. Graph coloring, which assigns labels to prevent conflicts, addresses scheduling problems like non-overlapping port operations. These constructs prepare students for various challenges, from sustainable ship design to global supply chains, meeting the industry's needs. By mastering abstract graph models, students contribute to maritime research, theorizing solutions for complex problems such as network vulnerability while ensuring rigorous theoretical education.

#### 4. Theoretical Applications in Maritime Transportation Networks.

Maritime transportation, which handles over 90% of global trade, depends on complex networks of ports, routes, and vessels that require solid theoretical frameworks for analysis. Graph theory, a key part of discrete mathematics, simplifies these systems into graphs, where vertices represent ports or waypoints and edges represent sea routes. This allows researchers at universities like Gdynia and Shanghai to explore structural properties like connectivity and resilience without needing empirical data. Directed graphs model one-way shipping lanes affected by currents, while weighted graphs address abstract costs like distance or environmental impact, supporting studies of efficiency. This section discusses how graph-theoretic concepts – connectivity, cycles, centrality, domination, and multilayer graphs – aid in analyzing maritime transportation networks, leading to insights about operational efficiency and weaknesses.

A maritime transportation network can be depicted as a graph  $G = (V, E)$ , where  $V$  stands for ports or navigational points and

$E$  represents sea routes. Directed graphs illustrate one-way relationships, such as lanes restricted by currents or regulations, while weighted graphs give abstract values to edges representing theoretical costs like transit time or fuel use. Multigraphs allow for multiple edges between vertices, modeling parallel routes and enhancing theoretical representations. For instance, a weighted digraph can illustrate a trade network, enabling researchers to theorize about route optimization without numerical data, in line with maritime education's emphasis on conceptual clarity.

Connectivity ensures a path exists between any two vertices, modeling trade networks where all ports are theoretically reachable. A disconnected graph reveals vulnerabilities, such as isolated ports, and guides discussions on network strength. Cycles represent closed paths that return to their starting vertex and are essential for theorizing how to make systems fault-tolerant. For example, a cycle connecting ports ensures that alternative routes are available if a primary route becomes inaccessible, improving resilience in theoretical models of global trade. Maritime students can use cycles to think about backup routes, which deepens their understanding of network reliability.

Centrality measures identify key points in transportation networks. Degree centrality, which counts the number of edges connected to a vertex, highlights significant ports like Singapore that have many connections. Betweenness centrality points out bottleneck nodes, like straits, that appear on various shortest paths. These ideas help scholars theorize about prioritizing key nodes for resource distribution, which can enhance network efficiency conceptually. Domination models the minimum placement of resources so that every vertex is adjacent to a chosen node, representing the strategic positioning of monitoring stations to cover a network. In maritime settings, this supports theoretical security models, ensuring comprehensive coverage with minimal resources.

Multilayer graphs combine different types of networks, with physical routes as one layer of edges and communication links as another. The edges across layers represent interactions, such as communications between vessels and ports. This approach allows researchers to think about how disruptions in communication can impact transport efficiency, leading to a comprehensive understanding of complex trade systems. By utilizing these concepts – connectivity, cycles, centrality, domination, and multilayer graphs – maritime universities develop strong and efficient transportation networks, preparing students and researchers to tackle global trade challenges through theoretical frameworks.

#### 5. Theoretical Applications in Naval Architecture.

Naval architecture, a fundamental discipline at maritime universities such as Gdynia Maritime University and Shanghai Maritime University, deals with the design, construction, and maintenance of ships and needs solid theoretical frameworks to model complex subsystems and processes. Graph theory, a basic tool in discrete mathematics, represents ship subsystems – like electrical, structural, or communication systems – as graphs, in

which vertices stand for components and edges denote connections. This setup enables scholars to theorize about system reliability and efficiency without requiring empirical data. Additionally, graph theory can depict design processes as graphs, in which vertices represent tasks and edges represent dependencies, aiding in the theoretical optimization of workflows. This section discusses how graph-theoretic concepts – connectivity, betweenness centrality, cycles, directed acyclic graphs (DAGs), and planar graphs – support theoretical inquiry in naval architecture, leading to insights about resilient ship designs and efficient design processes.

Ship subsystems can be represented as a graph  $G = (V, E)$ , where  $V$  denotes components (such as generators or structural joints) and  $E$  represents connections (like wiring or supports). Directed graphs capture one-way relationships, such as power flows in electrical systems. Weighted graphs provide abstract values to edges that reflect metrics like resistance or load capacity. Multigraphs model redundant connections, such as parallel wiring that ensures system reliability. For example, an electrical network graph allows researchers to theorize about how to improve power distribution without numerical data. Connectivity guarantees that all components are linked, modeling robust subsystems where a path exists between any pair of vertices. A disconnected graph points out vulnerabilities, such as an isolated structural joint, guiding discussions about system resilience.

Betweenness centrality highlights key nodes in a network, like important electrical hubs or load-bearing joints, that are crucial for many shortest paths and may fail under stress. Theorizing ways to reinforce these nodes boosts system reliability, which is vital in naval architecture education. Cycles represent backup pathways, like redundant circuits in electrical systems or extra supports in structures, which are essential for theorizing fault-tolerant designs. For example, a cycle in a structural network ensures stability under stress, providing insights into strong ship designs.

Graph theory also models the ship design process as a directed acyclic graph (DAG). In this model, vertices represent tasks (like hull design or propulsion integration) and edges illustrate dependencies, ensuring logical workflows. The topological ordering of a DAG optimizes task sequences and reduces theoretical bottlenecks, such as ensuring that structural design is completed before other steps. Planar graphs, which can be drawn without edges crossing, model ship deck layouts to optimize space and accessibility. Bipartite graphs, where vertices are divided into two groups, represent interactions between component types, like mechanical and electrical systems, aiding in theorizing their integration. These concepts allow students to imagine efficient design processes and subsystem configurations, in line with naval architecture's focus on reliability and efficiency.

By using connectivity, betweenness centrality, cycles, DAGs, and planar graphs, maritime universities develop resilient and effective ship designs. These frameworks enable researchers and students to investigate structural properties and theorize solutions to issues such as system failures or design inefficiencies, preparing them to tackle complex naval architecture challenges

through careful, abstract reasoning grounded in graph theory.

## 6. Theoretical Applications in Shipping Logistics.

Shipping logistics is an important field at maritime universities like Gdynia Maritime University and Shanghai Maritime University, overseeing global supply chains and routing systems that facilitate the transport of over 90% of global trade by sea. Graph theory, an essential part of discrete mathematics, illustrates these systems as graphs where vertices represent entities like ports, warehouses, or suppliers, and edges represent transport links like sea routes or intermodal connections. This approach allows researchers to theorize about efficient resource flows and connections without using empirical data, leading to insights into logistics challenges. This section discusses how graph-theoretic concepts – weighted graphs, multilayer graphs, shortest paths, minimum spanning trees, domination, and bipartite graphs – aid in the theoretical analysis of shipping logistics, enhancing education and research in maritime universities.

A shipping logistics system can be represented as a graph  $G = (V, E)$ , where  $V$  stands for entities (like ports or distribution centers) and  $E$  denotes transport links (like shipping routes or rail connections). Directed graphs depict one-way relationships, such as routes restricted by trade regulations, while weighted graphs assign abstract values to edges to represent theoretical costs like transit time or environmental impact. Multigraphs model parallel transport options, such as various shipping services between ports, enhancing theoretical insights. For instance, a weighted digraph can represent a supply chain, helping researchers theorize about reducing abstract costs, in line with the focus on conceptual rigor in maritime education.

Weighted graphs play a crucial role in exploring efficient resource flows. Edge weights, representing abstract measures like time or fuel use, enable studies of optimization, such as finding ways to cut transit costs in a supply chain. In maritime courses, students use weighted graphs to envision cost-effective logistics setups, fostering analytical skills. Multilayer graphs combine different transport modes, such as sea, rail, and road, with vertices signifying hubs and cross-layer edges representing transfers, such as cargo moving from ships to trains. This framework permits researchers to theorize about seamless connectivity across various modes, addressing complex logistics systems conceptually.

Shortest paths illustrate efficient cargo routes, such as from a supplier to a destination port. Minimum spanning trees (MSTs), which are acyclic subgraphs connecting all vertices with the least edge weight, show efficient network structures – like linking all ports in a trade region while minimizing infrastructure requirements. These concepts allow students to investigate theoretical optimization strategies that prepare them for logistics challenges. Domination models the placement of minimal resources, where a dominating set guarantees every vertex is adjacent to a chosen node, reflecting strategic warehouse placements to cover a network. In maritime contexts, this supports theoretical models for efficient resource distribution.

Bipartite graphs, where vertices are separated into two sets (like suppliers and ports), illustrate interactions such as cargo

assignments, theorizing effective matching strategies. Graph coloring, which assigns labels to avoid conflicts, represents scheduling issues like ensuring non - overlapping delivery times at ports. These concepts provide a versatile toolkit for maritime researchers to conceptualize logistics systems, fostering innovation in theory. By applying weighted graphs, multilayer graphs, shortest paths, MSTs, domination, and bipartite graphs, maritime universities explore efficient logistics models and prepare students and scholars to take on global supply chain challenges through effective abstract reasoning.

## 7. Challenges and Theoretical Extensions.

Graph theory offers a strong framework for modeling maritime systems, such as transportation networks and ship subsystems, by turning entities like ports and components into vertices and connections into edges [5]. However, static graph models have limitations: they rely on fixed structures and unchanging edge properties, which do not capture the dynamic and uncertain nature of maritime environments. Factors like weather, regulations, and trade fluctuations affect connectivity [16]. These limitations challenge the use of traditional graphs in maritime education and research and require theoretical extensions to address change, uncertainty, and complexity. This section looks at these challenges and suggests extensions – temporal graphs, fuzzy graphs, and hypergraphs – to improve the theoretical modeling of maritime systems at universities like Gdynia and Shanghai, supporting strong conceptual frameworks.

Static graphs assume fixed relationships, which makes them unable to represent time-varying maritime networks, such as shipping routes disrupted by seasonal storms or regulatory changes [17]. For example, a static graph modeling a trade network cannot reflect temporary route closures, limiting its capacity to theorize resilience under dynamic conditions [7]. Temporal graphs solve this issue by adding time as a dimension: edges exist within specific intervals, allowing models to show changing relationships [6]. In a temporal graph, a sea route might be active only during certain times, letting researchers think about adaptive network structures, such as rerouting strategies, even without empirical data [17]. This approach supports theoretical studies of network strength in maritime transportation, aligning with the theoretical goals of maritime curricula.

Uncertainty in maritime systems, like changing route reliability due to weather or mechanical issues, poses another challenge for rigid graph models, which assume predictable connections [5]. Fuzzy graphs provide a theoretical solution: they assign membership degrees (values between 0 and 1) to edges, showing partial connectivity [13]. For instance, a fuzzy edge between ports could reflect abstract reliability, enabling scholars to model uncertain trade routes conceptually [14]. In maritime logistics, fuzzy graphs support theoretical analysis of supply chain flexibility, allowing the hypothesis of robust configurations under different conditions without needing numerical data [16]. This method helps students build analytical skills, preparing them to handle uncertainty in maritime research.

The complexity of maritime networks involves interactions among many entities, like port alliances and intermodal logis-

tics, which go beyond the capabilities of pairwise edge models [10]. Hypergraphs generalize edges to connect multiple vertices, capturing more complex relationships [19]. For example, a hyperedge could represent a multi-port consortium, theorizing about collaborative trade structures such as those in global shipping alliances [20]. In naval architecture, hypergraphs help model subsystem interactions, like blended electrical and structural dependencies, enabling conceptual analysis of system integration [5]. This abstraction supports research across different disciplines, addressing complex maritime issues theoretically on the basis of graph theory.

These extensions – temporal graphs for change, fuzzy graphs for uncertainty, and hypergraphs for complexity – improve graph theory's usefulness in maritime education and research. Temporal graphs allow students to think about adaptive networks, fuzzy graphs help theorize under uncertainty (which pervades most maritime settings), and hypergraphs model intricate interactions. This aligns with the industry's need for strong frameworks in graph theory [16]. However, challenges still exist, such as defining temporal intervals or fuzzy membership functions, requiring further theoretical refinement [7]. Maritime universities can incorporate these extensions into their curricula, encouraging rigorous analysis of dynamic, uncertain, and complex systems. This prepares students and researchers to tackle global maritime challenges with innovative concepts.

## Conclusions.

Graph theory, a key part of discrete mathematics, provides a solid theoretical framework for maritime universities, in particular institutions such as Gdynia Maritime University and Shanghai Maritime University. It allows the abstraction of complex systems into vertices and edges to study connectivity, efficiency, and resilience [5,17]. By including graph theory in curricula, maritime education enhances analytical skills, preparing students to model systems like port networks, ship subsystems, and supply chains conceptually [7]. In discrete mathematics and operations research courses, students learn to understand maritime challenges using ideas like cycles and centrality, preparing them to engage with global trade complexities, where over 90% of goods are transported by sea [16]. In research, graph theory supports theoretical modeling of transportation networks, naval architecture, and logistics, hypothesizing solutions to challenges like network vulnerabilities even without empirical data [5,9].

Despite its strengths, static graph models struggle to capture the dynamic, uncertain, and complex aspects of maritime systems [17]. Theoretical extensions – temporal graphs for time-varying networks, fuzzy graphs for uncertainty, and hypergraphs for interactions among multiple entities – address these challenges and expand the conceptual flexibility of maritime research [6,13,19]. Temporal graphs showcase changing relationships, such as seasonal route adjustments, while fuzzy graphs express uncertain connections, like variable route reliability [14]. Hypergraphs illustrate complex interactions, such as port alliances, promoting interdisciplinary insights [20]. These

extensions, when added to curricula, prepare students to theorize about adaptive and robust maritime systems, aligning with the industry's need for sustainable solutions [16].

The theoretical focus of graph theory ensures that maritime universities stay at the cutting edge of addressing global maritime challenges. By focusing on abstract ideas rather than computational methods, graph theory builds strong analytical frameworks, enabling researchers and students to explore foundational principles before handling practical applications [7]. Future research can further develop these extensions, refining temporal and fuzzy models to enhance theoretical strength and support sustainable ocean economies [16]. Graph theory thus empowers maritime education and research, encouraging innovative solutions to the complexities of global trade, naval architecture, and logistics through strong conceptual work.

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